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THE SERVICE INDUSTRIES IN RELATION TO EMPLOYMENT TRENDS[1]

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I

THE AVAILABLE FIGURES on persons with gainful occupations show a steady decline since 1870 in the proportion attached[2] to the group of industries which includes agriculture, forestry and fishing, manufactures, extraction of minerals, and construction. Conversely, the proportion has steadily increased in the group of industries which includes transportation and communication, trade and finance, professional service, domestic and personal service, and public service. (See Tables 1 and 2.)

Most of the difference in trends between these two groups of industries is attributable to the declining relative importance of agricultural pursuits. If we exclude agriculture, the distribution of persons between these two groups of industries (which we shall from now on call "commodity-producing" and "service" industries, respectively) remained about equal throughout the period from 1870 to 1920. But after 1920, even if agriculture is excluded from the picture, the shifts in the industrial pattern of the country resulted in a definite increase in the proportion of persons attached to the services. (See Table 2.)

Although the figures on gainful persons include both the employed and the unemployed, the trends in their industrial composition reflect also the employment trends in the several industry groups since the attachments of individuals to particular industries are, over long periods, likely to respond to the trend of employment opportunities in those industries. The employment opportunities, in turn, are conditioned by the trends of production and of labor productivity.

[1] This is an abbreviated version of a paper presented on December 27, 1939 at the Econometric Society's meeting in Philadelphia, Pennsylvania. The authors wish to acknowledge their indebtedness to the following members of the National Research Project's staff: Daniel Carson from whose study of labor supply and employment the basic data were drawn, Max Lipowitz and Henrietta Liebman who aided in the investigation of several of the problems discussed; Mr. Lipowitz also helped to prepare the statistical data used in this paper.

[2] The terms "number of persons with gainful occupations" and "number of persons attached" as used in this paper are synonymous with the Census of Population's "gainfully occupied." The number of persons attached to an industry include the employed and unemployed, wage and salary earners, and entrepreneurs. The term "attached to an industry" more specifically signifies the number of persons who consider themselves usually attached to that industry.

During the 1920's the demand which the growth of production in the commodity-producing industries made on labor was to a very large extent offset by increased productivity. Despite a more than 50-percent increase in production between 1919 and 1929, the manufacturing industries increased their employment by only about 2 per cent.[3] The

TABLE 1

INDUSTRIAL DISTRIBUTION OF PERSONS WITH GAINFUL OCCUPATIONS, DECENNIALLY, 1870–1930[a]

(In thousands)

	1870	1880	1890	1900	1910	1920	1930
Population[b]	39,818	50,156	62,622	75,995	91,972	105,711	122,775
Total persons with gainful occupations	12,925	17,392	23,453	29,073	37,371	41,614	49,155
Total excluding agriculture	6,121	8,712	13,428	18,015	25,773	30,498	38,671
Commodity-producing industries	9,951	13,264	16,830	20,003	24,327	26,154	28,003
Commodity-producing industries excluding agriculture	3,147	4,584	6,805	8,944	12,730	15,037	17,519
Agriculture	6,804	8,680	10,025	11,058	11,597	11,117	10,484
Forestry and fishing	60	94	178	200	247	284	269
Extraction of minerals	199	326	483	766	1,061	1,238	1,156
Manufacturing	2,192	3,328	4,753	6,385	8,402	11,091	11,401
Construction	696	836	1,391	1,594	2,307	2,172	3,030
Industry not specified	....	....	....	....	713	252	1,663
Services	2,973	4,128	6,623	9,071	13,044	15,460	21,152
Transportation and communication	427	618	1,161	1,713	2,840	3,678	4,071
Trade and finance	855	1,276	2,087	2,959	4,075	5,012	7,530
Domestic and personal service	1,254	1,511	2,242	2,823	3,771	3,393	4,815
Professional service, recreation and amusement	215	330	549	754	1,114	1,496	2,246
Public service	223	392	583	821	1,244	1,882	2,490

[a] Source: Daniel Carson assisted by Henrietta Liebman, *Labor Supply and Employment, Preliminary Statement of Estimates Prepared and Methods Used*, WPA National Research Project, mimeo., Nov. 1939. The data for this report are based on the Decennial Census of Population. The figures for all years have been classified on an industrial basis and are comparable to the classification in the *Fifteenth Census of the United States: 1930*, "Population," U. S. Dept. Com., Bur. Census, 1933, Vol. V, Chapter 7.

Some changes have been made in the Census classification. The major changes involve: (1) the setting up of a new category, "construction," by combining building construction (classified by the Census under manufacturing and mechanical industries) with construction and maintenance of streets and roads (originally classified under transportation and communication); (2) the shifting of public school teachers, secretaries, clerks, and janitors, from professional service to public service; (3) the shifting of postal service from transportation and communication to public service; and (4) the shifting of automobile repair shops from manufacturing to transportation and communication.

In addition to changes in classification, the following adjustments have been made to the over-all totals: (1) the total for 1890 includes an adjustment of 135,175 for the undercount of male youth in addition to the correction made by the Census; (2) the total for 1930 includes an adjustment of 325,000 persons for omission of new workers by the Census; and (3) the totals for 1870 and 1910 conform to the totals in *Industrial Distribution of the Nation's Labor Force: 1870 to 1930*, Dept. of Commerce, Bureau of the Census, processed release dated October 23, 1938.

[b] Data for: 1870 from *Fifteenth Census of the United States: 1930*, "Population," U. S. Dept. Com., Bur. Census, 1933, Vol. II, p. 32, includes a correction for undercount of population; 1880–1930, from *ibid.* Vol. V, p. 37.

Note: All figures were rounded after computations were made.

number of persons engaged in agricultural pursuits remained stationary during the same period,[4] while the number of those employed in the

[3] See Harry Magdoff, Irving H. Siegel, and Milton B. Davis, *Production, Employment and Productivity in 59 Manufacturing Industries, 1919–36*, WPA National Research Project, Report No. S-1, May, 1939, Part One, p. 65.

[4] Between 1919 and 1929 employment on farms (farm operators, family labor, and hired labor) remained practically constant; production in this ten-year period increased approximately 15 per cent. See Eldon E. Shaw and John A. Hopkins, *Trends in Employment in Agriculture, 1909–36*, WPA National Research

extraction of minerals declined 4 per cent.[5] Of the commodity-producing industries, only construction showed sizable increases in employment. (See Table 3.)

Of the increase of approximately 7 million in total employment between 1920 and 1929 only about 2½ million are attributable to the com-

TABLE 2

PERCENTAGE DISTRIBUTION OF PERSONS WITH GAINFUL OCCUPATIONS, DECENNIALLY, 1870–1930[a]

	1870	1880	1890	1900	1910	1920	1930
A. Total, all industries	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Commodity-producing industries	77.0	76.2	71.8	68.8	65.1	62.9	56.9
Agriculture	52.6	49.9	42.7	38.0	31.0	26.7	21.3
Forestry and fishing	0.5	0.5	0.8	0.7	0.7	0.7	0.5
Extraction of minerals	1.5	1.9	2.1	2.6	2.8	3.0	2.3
Manufacturing	17.0	19.1	20.3	22.0	22.5	26.7	23.2
Construction	5.4	4.8	5.9	5.5	6.2	5.2	6.2
Industry not specified	....	....	....	....	1.9	0.6	3.4
Services	23.0	23.8	28.2	31.2	34.9	37.1	43.1
Transportation and communication	3.3	3.6	4.9	5.9	7.6	8.8	8.3
Trade and finance	6.6	7.3	8.9	10.2	10.9	12.0	15.3
Domestic and personal service	9.7	8.7	9.6	9.7	10.1	8.2	9.8
Professional service, recreation and amusement	1.7	1.9	2.3	2.6	3.0	3.6	4.6
Public service	1.7	2.3	2.5	2.8	3.3	4.5	5.1
B. Total, all industries excluding agriculture	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Commodity-producing industries excluding agriculture	51.4	52.6	50.7	49.6	49.5	49.3	45.3
Services	48.6	47.4	49.3	50.4	50.5	50.7	54.7

[a] Computed from Table 1. The percentages were computed prior to the rounding off of the figures in Table 1.

modity-producing industries while 4½ million are ascribable to the service industries and occupations. (See Table 3.) The difference in the rates of growth between the two groups of industries appears even more striking when it is remembered that the 2½-million increase in the commodity-producing industries amounts to only 10 per cent of their total employment in 1920, while the 4½-million increase in the service industries amounts to almost 40 per cent of their total employment in 1920. In view of the persistence of large volumes of unemployment

Project, Report No. A-8, November, 1938, p. 11; and Raymond G. Bressler, Jr. and John A. Hopkins, *Trends in Size and Production of the Aggregate Farm Enterprise, 1909–36*, WPA National Research Project, Report No. A-6, July, 1938, p. 20.

[5] In the mineral extractive industries production increased 39 per cent between 1919 and 1929 while average annual wage-earner employment decreased by 4 per cent. Data were obtained from Vivian E. Spencer, *Production, Employment and Productivity in Mineral Extraction*, WPA National Research Project, 1940. This report includes the Cement and the Petroleum and Natural Gas industries in addition to the industries covered by the *Census of Mines and Quarries*, U. S. Dept. Com., Bur. Census. The increase in employment between 1919 and 1929 in the Petroleum and Natural Gas industry offsets materially the 16-per-cent decline shown by the *Census of Mines and Quarries*.

TABLE 3

ANNUAL ESTIMATES OF EMPLOYMENT OF WAGE AND SALARY EARNERS, 1920–1937[a]

(In thousands)

	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937
Total commodity-producing industries and services[b]	35,879	33,013	34,898	37,553	37,362	38,881	40,293	40,646	41,231	42,936	40,509	37,238	33,656	33,803	35,934	37,439	39,242	40,941
Total excluding agriculture	24,517	21,601	23,455	26,168	26,000	27,435	28,759	29,400	29,936	31,647	29,336	26,079	22,587	22,780	25,082	26,267	28,245	30,111
Commodity-producing industries[b]	24,335	22,111	24,333	25,014	24,525	25,361	25,995	25,861	26,113	26,805	24,984	22,828	20,484	20,918	22,172	23,227	24,290	25,214
Commodity-producing industries excluding agriculture	12,973	10,699	11,990	13,629	13,163	13,915	14,461	14,615	14,818	15,516	13,811	11,669	9,415	9,895	11,320	12,055	13,293	14,384
Agriculture[b]	11,362	11,412	11,443	11,385	11,362	11,446	11,534	11,246	11,295	11,289	11,173	11,159	11,069	11,023	10,852	11,172	10,997	10,830
Forestry and fishing	206	178	229	238	231	232	224	210	206	212	170	114	90	108	130	137	146	168
Extraction of minerals	1,118	849	817	1,084	986	968	1,051	984	927	958	882	756	619	635	746	753	794	830
Construction	1,682	1,865	2,169	2,133	2,267	2,517	2,604	2,791	2,843	2,690	2,340	1,893	1,191	1,022	1,153	1,291	1,605	1,673
Manufacturing	9,530	7,362	8,212	9,458	8,893	9,287	9,542	9,485	9,592	10,256	9,062	7,725	6,527	7,127	8,175	8,711	9,482	10,356
Industry not specified	437	445	563	716	786	911	1,040	1,145	1,250	1,400	1,357	1,181	988	1,003	1,116	1,163	1,266	1,357
Services	11,544	10,902	11,465	12,539	12,837	13,520	14,298	14,785	15,118	16,131	15,525	14,410	13,172	12,885	13,762	14,212	14,952	15,727
Transportation	2,788	2,447	2,502	2,838	2,828	2,896	3,049	2,991	2,955	3,028	2,796	2,418	2,044	1,967	2,070	2,114	2,322	2,440
Communication	383	385	408	441	454	468	488	486	500	548	540	477	435	390	395	400	417	453
Trade	2,700	2,565	2,757	3,008	3,149	3,280	3,451	3,639	3,706	3,946	3,879	3,558	3,161	3,154	3,411	3,429	3,575	3,763
Finance and property handling	488	549	597	764	804	849	990	1,001	1,034	1,046	909	900	853	768	788	789	814	839
Domestic and personal service	2,719	2,422	2,643	2,916	2,878	3,224	3,412	3,582	3,689	3,990	3,825	3,502	3,149	3,116	3,488	3,747	3,957	4,241
Professional service, recreation and amusement	846	867	881	912	959	1,015	1,065	1,124	1,202	1,475	1,391	1,358	1,361	1,367	1,415	1,438	1,443	1,509
Public service	1,621	1,666	1,677	1,660	1,765	1,788	1,842	1,962	2,032	2,099	2,185	2,197	2,169	2,123	2,194	2,295	2,424	2,482

[a] Source: Daniel Carson assisted by Henrietta Liebman, *Labor Supply and Employment, Preliminary Statement of Estimates Prepared and Methods Used*, WPA National Research Project, mimeo., November, 1939. The employment figures for 1920 and 1930 were obtained by successively subtracting from the total number of persons attached to an industry the number of entrepreneurs and the number unemployed. The number of persons attached to an industry and the number of entrepreneurs were computed from data in the *Census of Population*, "Occupations" (See Table 1, footnote a). The number unemployed in 1930 was obtained from *Fifteenth Census of the United States: 1930*, "Unemployment," U. S. Dept. Com., Bur. Census, 1931, Vols. I and II, and unemployment in 1920 was estimated from several appropriate sources. The annual data were computed from available industrial censuses, from reports of government and private agencies, and from other available sources, and were used as interpolating media for the 1920 and 1930 figures mentioned above.

It should be noted that since the definitions of the individual industries and services in this table are determined by the Census of Occupation's industrial classification, these employment figures will not correspond exactly with those shown in industrial censuses (i.e., in the Census of Manufactures, Census of Mines and Quarries, etc.).

[b] The total for all industries and for commodity-producing industries will differ from the total shown in *Labor Supply and Employment* (*supra*) becasue of the difference in the figures for agriculture. Total agricultural employment as shown in this table includes family workers and farm managers and was computed from Eldon E. Shaw and John A. Hopkins, *Trends in Employment in Agriculture, 1909–1936*, WPA National Research Project Report No. A-8, November, 1938, p. 11. This National Research Project series was continued to 1937 in a release of the Bureau of Agricultural Economics, U. S. Department of Agriculture, dated January 13, 1939, dealing with farm wage rates, labor supply, and employment.

Note: All figures were rounded after computations were made.

since 1929, this continuous decline in the importance of commodity-producing industries (which in 1929 employed three-fifths of the total wage and salary earners) as absorbers of the country's growing labor supply has been centering increasing interest in the future role of the services as employers of labor.

The purpose of this paper is to examine the economic basis of the growth of the services by classifying and analyzing the diverse activities included in the so-called service industries according to the differences that may exist in the bases for their demand. The so-called service industries range from personal services supplied to ultimate consumers, to services which are supplied to the business and financial institutions which form part of the social organization of production, and to those services which represent mainly an extension of the process of producing commodities either by distributing them or by maintaining them in serviceable condition. A study of factors underlying employment trends in the services cannot therefore treat the service industries as if they constituted a homogeneous whole. The several components of the group as a whole have to be so classified as to fit the analytical framework determined by the purposes of the investigation.

Since the service industries produce in the main intangibles, the absence of a physical measure of production and the lack of any other standard unit of measurement make it exceedingly difficult, if not in most instances impossible, to evaluate the rates of growth of the several components of this group except in terms of the number of persons attached to an industry or, for more recent years, the number of persons employed. Moreover, the nature of the available data does not permit so rigid a classification of the heterogeneous activities represented by the so-called service industries as the purposes of this analysis require; many of the breakdowns made here are therefore dictated by the form in which the statistics are available and, since our analytical purposes required some regroupings of the available data, a certain amount of overlapping of categories was inevitable.[6] The classifications that will be used here represent a compromise between the theoretical needs of the analysis and the type of data available for measurement.

A study of the growth of the service industries as related to employ-

[6] The recent development of a uniform classification of industries and occupations by the interagency project sponsored by the Central Statistical Board should, in the future, enable easier adaptability of source data to analytical purposes. See Vladimir S. Kolesnikoff, "Standard Classification of Industries in the United States," *Journal of the American Statistical Association*, Vol. 35, No. 209, Part I, March, 1940, and Gladys L. Palmer, "The Convertibility List of Occupations and the Problems of Developing It," *ibid.*, Vol. 34, No. 208, December, 1939.

ment trends would require detailed qualitative and quantitative investigations of the nature and causes of growth in different segments of the services. This paper is intended to be only a first step in that direction. The value of the classifications that will be made here lies in a delineation of the differences in the several broad types of services as they relate to other types of economic activity and to the demand for employment. There are, for example, service industries in which the demand for labor is directly linked to the volume of commodities that is being produced or sold. Thus, the transportation of freight and the distributive trades embrace activities which constitute what are, in the main, extensions of the commodity-production processes. The volume of employment in financial and property-handling institutions, on the other hand, is related to the volume of savings and the opportunities for their investment. The employment trends of these institutions reflect in part the growth of the credit mechanisms and the growing complexity of the financial processes required for the management of the nation's productive resources, and in part the changes in the methods and techniques with which these activities are performed; in addition, there is a group of services which depends on the shifting about of claims to property and of monetary values based on purely speculative returns which bear little or no relationship to the business of producing and distributing.

Both commodity-handling functions and financial and property-handling functions are frequently carried on as an integral part of the commodity-producing industries. In such instances these service tasks are usually performed by clerical or professional workers who are attached directly to the commodity-producing industry. Since it was not possible to classify such employees according to their functions, we shall, in our subsequent discussion, have to handle this group separately and we shall evaluate their employment prospects in terms of those changes in the commodity-producing industries which affected them most directly.

The domestic and personal services represent still another type of economic activity. Their demand for labor depends directly on the volume of income available to final consumers, on the way in which that income is distributed, on changes in the consumption patterns of the community, and on the changes in the degree to which some of these services are becoming commercialized.

The public services, too, constitute a separate problem for analysis since they are subject to a set of factors peculiar to them alone. Their demand for labor is the result of public policy with relation to the personal needs of the population and the economic needs of the commercial and industrial community.

Our analysis thus starts out with five major groups of persons who are attached to service industries and occupations: (1) those attached to industries concerned with the handling of commodities, (2) those attached to financial and property-handling institutions, (3) salaried persons who are attached to the commodity-producing industries, (4) persons attached to the domestic, personal, and professional services, and (5) persons attached to the public services.

II

(1) The effective demand for labor in the first of our groups of services rests on the fact that they are engaged in the handling of commodities. This group includes those engaged in the transportation of raw materials and semifinished and finished goods, in communication, and in the distribution of commodities to the final consumer.

TABLE 4

NUMBER OF PERSONS ATTACHED TO TRANSPORTATION, COMMUNICATION, AND TRADE, 1910, 1920, AND 1930[a]
(In thousands)

	1910	1920	1930
Transportation	2,570	3,258	3,483
Steam railroads[b]	1,593	1,868	1,742
Electric railroads	185	220	195
Oil and gas pipe lines	4	12	25
Express companies	53	78	62
Waterborne transportation	222	280	300
Garages, greasing stations, auto laundries, and auto repair shops	49	367	682
Other transportation	465	435	477
Air	....	....	18
Trucks, transfer, and cab companies	325	395	437[c]
Livery stables	138	34	10
Other not specified	2	5	12
Communication	270	420	588
Telephone and telegraph	270	420	579
Radio broadcasting and transmitting	....	....	9
Trade	3,554	4,215	6,110
Wholesale and retail	3,504	4,078	5,852
Grain elevators, stockyards, and warehouses	49	105	108
Other trade	....	32	150
Advertising	....	....[d]	64
Other and not specified	....	....[d]	86

[a] Computed from Daniel Carson assisted by Henrietta Liebman, *Labor Supply and Employment, Preliminary Statement of Estimates Prepared and Methods Used,* WPA National Research Project, mimeo., November, 1939, based on *Fifteenth Census of the United States: 1930,* "Population," U. S. Dept. Com., Bur. Census, 1933, Vol. V, Ch. 2 and Ch. 7, and the *Thirteenth Census of the United States: 1910,* "Population," U. S. Dept. Com., Bur. Census, 1914, Vol. IV, Table 6. All series are comparable to the industrial classification of the 1930 Census as adjusted and reclassified in the above-mentioned National Research Project report.

[b] Includes part of the car and railroad shop category classified by the Census under manufacturing and mechanical industries.

[c] Exludes school-bus drivers (45,945) in 1930.

[d] Not given separately in 1920.

Note: All figures were rounded after computations were made.

In Table 4 we have summarized the statistics on the number of persons attached to the commodity-handling industries from 1910 to 1930. The number of persons attached to all transportation industries combined increased only 7 per cent between 1920 and 1930.[7] Such increases

[7] The available statistics do not permit the treatment of transportation in two separate parts—that supplying transportation to passengers for personal pur-

as took place were the result of the extensive increase in the use and servicing of automobiles, busses, and trucks for recreation, passenger and freight transportation, while on steam railroads productivity was rising faster than the demand for service with consequent declines in employment.[8]

The inclusion of the communication industries under the classification of handling of commodities is arbitrary to an extent. Communication, as in the case of transportation, includes—in addition to services directly supplying consumers' needs—services that are necessary to the production, transportation, and selling of goods. The communication services in the form of telephone, telegraph, and radio, are still relatively new and have been expanding rapidly during the last 20 years. As contrasted with transportation, which exhibited an over-all small increase in the number of persons attached between 1920 and 1930, the number of persons attached to communication increased about 40 per cent between these years. We find, however, that in the telephone industry, for which relevant data are available, the increases in employment during the twenties were considerably smaller than the increases in production. Thus, the index of employment in the telephone industry rose from 100 in 1917 to 164 in 1929, as compared with the increase from 100 to 182 in the index of production. By 1939, the index of production was 96 per cent of 1929, whereas the index of employment was only 75 per cent of 1929.[9]

The increase in the number of persons whose occupations are in wholesale and retail trade seems almost to have kept pace with the

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poses and that supplying passenger and freight transportation for business. It is important to note, however, that in steam railways—the largest segment of the transportation industry—the predominant source of revenue arises from the carrying of freight. Thus in 1929, almost 6 times as much revenue was obtained by steam railways from the carrying of freight as from the fares of passengers (*Statistical Abstract of the United States: 1937*, U. S. Dept. Com., Bur. For. and Dom. Com., 1938, pp. 380, 382).

[8] In the Class I Steam Railroads the volume of traffic (weighted revenue ton-miles plus passenger-miles) increased from 93.1 to 100 between 1919 and 1929 while employment decreased from 116.0 to 100 during the same period. Data from M. L. Jacobson, "Employment, Compensation, and Productivity of Railroad Labor," WPA National Research Project in co-operation with the Railroad Retirement Board, in preparation, and Witt Bowden, "Productivity, Hours and Compensation of Railroad Labor, 1933 to 1936," *Monthly Labor Review*, U. S. Dept. Labor, Bur. Lab. Stat., Vol. 45, No. 1, July, 1937, p. 80.

[9] See table on "Indexes of Production, Employment, and Productivity in the Telephone Industry, 1919–39," Exhibit No. 2750 submitted to the Temporary National Economic Committee in Washington, D. C., on April 26, 1940, in testimony presented by Corrington Gill and David Weintraub for the Work Projects Administration. Comparable data for 1917 were obtained from the *Census of Electrical Industries*, U. S. Dept. Com., Bur. Census.

ninefold expansion in the physical volume of production and sales which occurred between 1870 and 1930.[10] This phenomenon stands in sharp contrast to the fact that the number of persons attached to all commodity-producing industries in 1930 was less than three times as great as in 1870.

Wholesale and retail trades render services which require relatively little capital equipment. Their services can therefore be performed on a scale which ranges from the small jobber and the retailer who carries his wares on his back to the huge mail-order house or department store. The highly competitive conditions which prevail in these industries and the small amount of money which suffices to set up a store result in a rapid influx of new enterprisers who just as rapidly drop out again but who have meanwhile operated at a loss, have conducted an inefficient business, and thus contributed toward keeping down the level of productivity of the industry as a whole. In addition, the tendency toward an improvement in the quality of service rendered, frequently resulting in increased labor requirements, and the tendency toward improving the efficiency with which services are performed seem to be offsetting one another.

Since productivity in the commodity-handling functions of the distributive trades increased, if at all, much more slowly than in the commodity-producing industries, the increased volume of commodities handled has in the past constituted one source of absorption for the country's growing labor supply. Some of the persons absorbed must, however, be regarded as having assumed a status of disguised unemployment, judging from the higher rates of mortality of establishments engaged in retail trade and the incomes of large sections of the small businessmen.

[10] The number of persons attached to wholesale trade was estimated at 803,000 in 1870; 1,225,000 in 1880; 1,925,000 in 1890; 2,606,000 in 1900; 3,504,000 in 1910; 4,078,000 in 1920 and 5,852,000 in 1930. This series, based on data in the Decennial Census of Population, "Occupations," is comparable for all years to the industrial classification of wholesale and retail trade (including automobile agencies, stores, and filling stations) as presented in *Fifteenth Census of the United States: 1930*, "Population," U. S. Dept. Com., Bur. Census, 1933, Vol. V, pp. 568–72.

For the corresponding years during the period 1870–1930, the index of physical volume of production (with 1920 taken as 100) stood at 14, 24, 33, 48, 70, 100, and 125. This index was computed by splicing Persons' index of total production for 1870–1900 (Warren M. Persons, *Forecasting Business Cycles*, New York, John Wiley and Sons, Inc., 1931, p. 170) with the Department of Commerce index of physical volume of goods marketed at wholesale for 1900–1930 (*Survey of Current Business*, U. S. Dept Com., Bur. For. and Dom. Com., September, 1939, p. 11). The latter index included approximately 97 per cent of the total value of produced and imported goods in 1933 and 1935 (see *ibid.*, May, 1936, p. 16, footnote 2).

(2) The second of our service-industry groups embraces the financial and property-handling institutions. The accumulation of funds available for investment as well as the development of industrial techniques whose most effective utilization required an ever-increasing volume of capital, has been accompanied by a growth in the importance of these institutions and by an increasingly complex financial structure whose relations to the productive apparatus of industry have steadily grown more pervasive. Both of these factors are historically reflected, on the one hand, in the expansion of credit and banking activities, in the increasingly corporate character of industry, and in the skyrocketing of stock-market activity and, on the other hand, in the rapid increase in the number of persons engaged in occupations directly related to these activities.

Some of these functions are carried on by salaried employees directly attached to the commodity-producing industries and some by persons employed in the commodity-handling and other commercial services, but in the main they have assumed the form of independent business institutions.

TABLE 5

NUMBER OF PERSONS ATTACHED TO FINANCIAL AND PROPERTY-HANDLING INSTITUTIONS, 1910, 1920, AND 1930[a]

(In thousands)

	1910	1920	1930
Financial and property-handling institutions	521	797	1,420
Banking and brokerage	213	391	625
Insurance	155	227	507
Real estate	153	179	288

[a] Computed from Daniel Carson assisted by Henrietta Liebman, *Labor Supply and Employment, Preliminary Statement of Estimates Prepared and Methods Used*, WPA National Research Project, mimeo., November, 1939, based on *Fifteenth Census of the United States: 1930*, "Population," U. S. Dept. Com., Bur. Census, 1933, Vol. V, Ch. 2 and Ch. 7, and the *Thirteenth Census of the United States: 1910*, "Population," U. S. Dept. Com., Bur. Census, 1914, Vol. IV, Table 6. All series are comparable to the industrial classification of the 1930 Census.

Table 5 shows the number of persons who considered themselves attached to the financial and property-handling institutions for the years 1910 to 1930. The number of persons attached increased very markedly between 1920 and 1930 and at a faster rate than between 1910 and 1920. The increased rate of growth in the twenties, as compared with the prior decade, reflects primarily the marked increase in insurance and real-estate activities. To analyze the changes, we have divided the group as a whole into three subgroups: banking and brokerage, insurance, and real-estate activities.

There were a number of factors which tended to reduce labor requirements in the performance of banking activities. There was, for instance, a continuous concentration of banking activities in fewer units, thus making possible an increase in the efficiency of handling the available volume of transactions. There was also, in banks as well

as in insurance companies and large real-estate offices, an increase in the mechanization of office work and with it an increasing division of labor, increasing speed of operations, standardization of office forms and of methods of work, and an improvement of the methods of management and supervision of office work.[11] Though the number of office employees tended to increase with mechanization because "office machines make it possible to carry out . . . operations which would be too expensive if they had to be done by hand,"[12] the introduction of such machines also tended to "curb the rapid rise in number of employees with the increased office function of modern business."[13] On the other hand, there were forces at work, for example the expansion of stock-market activities, investment trusts, and consumer credit, which tended to increase the volume of employment.

In the field of insurance, total assets of all insurance companies increased 16-fold between 1895 and 1930.[14] In spite of the rapid growth of fire, marine, casualty, surety, and other types of insurance, life insurance remained the most important type. Since 1890 the amount of life insurance in force increased almost 25 times as fast as did the population in the United States. In essence, life insurance represents a unique form of savings institution where large volumes of savings are concentrated and invested in the economic activities of the nation. The ratio of life-insurance companies' income to total national income increased from 0.1 per cent in 1880 to 6.7 per cent in 1930. During this period an increasing proportion of the income of life-insurance companies came from their investments rather than from premiums.[15]

Of the total employment by insurance carriers, agencies, and brokerage offices in 1935, about 47 per cent were employed by life-insurance carriers, 16 per cent by casualty-, surety-, and miscellaneous-insurance carriers, 10 per cent by fire- and marine-insurance carriers, and about 26 per cent by general insurance agencies and brokerage offices. Of the total personnel in insurance carriers' offices in 1935, almost 50 per cent

[11] "The Use of Office Machinery and Its Influence on Conditions of Work for Staff," *International Labour Review*, Vol. 36, No. 4, October, 1937, pp. 486–516.

[12] *Ibid.*, p. 515.

[13] Ethel Erickson, "The Employment of Women in Offices," *Women's Bureau Bulletin*, U. S. Department of Labor, Bulletin No. 120, pp. 15–17.

[14] In 1895 the admitted assets of life-, fire-, marine-, casualty-, surety-, and miscellaneous-insurance companies totalled roughly 1½ billion dollars. By 1930 this figure had jumped to almost 25 billion dollars. Data are from annual issues of the *Insurance Yearbook*, Spectator Company, Philadelphia, Pa.

[15] See testimony of Dr. Donald H. Davenport in *Investigation of Concentration of Economic Power*, U. S. Congress, Senate, Hearings before the Temporary National Economic Committee on Pub. Res. No. 113 (75th Cong.), 76th Cong., 1st sess., Feb. 6–10 and 14–17, 1939, Part 4, "Life Insurance," pp. 1165–97.

were office and clerical employees, 20 per cent direct selling employees, 20 per cent office solicitors, and 4 per cent executives and salaried corporation officers.[16] The large number of clerical employees who are concentrated in the home offices of the carriers represent a promising field for mechanical and managerial changes which tend to reduce the labor requirements and any evaluation of employment prospects in the field of insurance must take this factor into account as well as the prospects for the further growth of insurance.

Of the three subgroups which we have set up, real-estate activities showed the smallest increase in the number of persons attached between 1910 and 1930. The number attached to real estate increased 17 per cent between 1910 and 1920 compared with an increase of 83 per cent in banking and brokerage and 46 per cent in insurance. (See Table 5.) The probable reason for the relatively small increase in the number of persons attached to real estate during the period 1910 to 1920, is the fact that construction activity was very low during most of that period.[17] But between 1920 and 1930, the number of persons attached to real estate increased as rapidly as did banking and brokerage but not so rapidly as did insurance. This increase was no doubt influenced to the greatest extent by the boom in construction during the middle twenties and the increase in the volume of real-estate trading that accompanied that boom. Another important factor is the change in the relative importance of certain functions performed by real-estate agencies and their assumption of new functions.

The three subgroups, banking and brokerage, insurance, and real estate, carry on a multitude of services supplied to individuals, to each other, and to other businesses. An evaluation of employment prospects in this group of services would require a detailed investigation of the changing importance and economic role of the different segments of the group. Whether they grow at a faster or slower rate than the rest of the economy depends upon such factors as changes in the functions undertaken and performed by these institutions, changes in the financial and industrial structure of the economy, changes in the size of enterprises, and changes in the distribution of income and in the proportions of income saved and invested. What is important in the context of this paper, however, is that since these services deal with the industrial, commercial, and financial activities of the community

[16] *Census of Business: 1935*, "Insurance," U. S. Dept. Com., Bur. Census, pp. ix, xiv, 4, 12, and 15.

[17] The annual building permits per capita were declining sharply between 1910 and 1918. See John R. Riggleman, "Building Cycles in the United States, 1875 to 1932," *Journal of the American Statistical Association*, Vol. 28, No. 182, June, 1933, p. 178.

through the management of the savings and investments of industry and of individuals, their growth and the growth of employment in these activities depend ultimately on the growth of the national income and on the character of that growth.

(3) Our third group consists of the salaried employees who are attached to the commodity-producing industries. The advancing technology of commodity production and the increasing centralization of ownership required not only more technical and clerical workers, whose functions aided and facilitated the production processes, but also more and more work became necessary to distribute the increased volume of commodities and to facilitate the increasingly complex financial arrangements and transactions.

Advancing technology and the accompanying growth of large-scale production have greatly enhanced the need for superintendents, foremen, technicians, and office personnel. The specialization of production by departments and operations, the separation of tasks, the growing need for efficient controls over the flow of work, all required an increasing volume of technical and clerical assistance. The office controls necessary in large plants required relatively larger office staffs than did small-scale production, where very often the management of processes, hiring and firing, and the instruction and training of workmen were directly in the hands of the owner of the business.

The increasing concentration of ownership through the substitution of managers and officials for owner control contributed toward the growth of salaried employees by a division of labor in management.[18] Moreover, growing concentration of ownership meant increasingly complex financial structures and a growing number of clerical employees. An increasing amount of effort also had to be devoted to the stimulation of mass markets for the output of mass production and to the distribution of goods. Many of these increased entrepreneurial functions were delegated by the owners to hired salaried workers, thus contributing to the growth of advertising technicians, sales personnel, and related employment.

[18] For example, of the 46,500 manufacturers, officials, and managers listed in the 1870 Census of Occupations, 44,500 were manufacturers and only 2,000 were officials and managers. Between 1870 and 1910 the number of manufacturers increased fivefold, but there were about sixty times as many officials and managers in 1910 as in 1870. Between 1910 and 1930, the number of manufacturers decreased by 20 per cent and the number of officials and managers doubled. Data for: 1870–1900 from *Twelfth Census of the United States: 1900*, "Special Reports," U. S. Dept. Com. and Labor, Bur. Census, 1904; "Occupations," pp. xlviii-xlix; 1910–30 from *Fifteenth Census of the United States: 1930*, "Population," U. S. Dept. Com., Bur. Census, 1933, Vol. V, Ch. 2, as reclassified in *Labor Supply and Employment* (*supra*), see Table 1, footnote a.

In the manufacturing industries, for example, an increase of 36 per cent between 1909 and 1929 in the number of wage earners attached was accompanied by an increase of 98 per cent in the number of salary earners.[19] Similar trends can be observed in the mining industries.[20]

However, as large-scale production and its concentration increased in importance, the growth in demand for salaried employees slowed down. Thus, while the increase of about 50 per cent in manufactured production from 1899 to 1909 brought an increase in salaried workers of 117 per cent, the more than 50-per-cent increase in production between 1919 and 1929 was accompanied by an increase of only 9 per cent in the number of salaried employees.[21]

(4) Our fourth group of service industries includes the domestic and personal services and the occupations that are concerned with the health, religion, recreation, and amusement of the community.

In the main, this group of services supplies the needs of final consumers. In this its economic function is similar to that of the consumers'-goods industries. Likewise, the demand for both of these types of activities is based on the consumers' purchasing power, which in turn depends upon the size of the national income and the way it is distributed, and on the consumption patterns of the various income groups. There are, however, important differences between the consumers'-service industries and the consumers'-commodity industries with respect to the character of employment which they offer and the demand for labor which they constitute.

[19] Wage earners increased from 6,473,000 in 1909 to 8,839,000 in 1929 while the number of salary earners rose for the corresponding years from 790,000 to 1,567,000. Data are from *Fifteenth Census of the United States: 1930*, "Manufactures: 1929," U. S. Dept. Com., Bur. Census, 1933, Vol. II, p. 15. Data adjusted to include employees in central administrative offices.

[20] In extraction of minerals, the decline in the number of wage earners from 931,000 to 788,000 between 1909 and 1929 was accompanied by an increase from 38,000 to 49,000 in the number of salary earners. The net result is reflected in an increase of from 3.9 in 1909 to 5.8 in 1929 in the per cent of the total wage and salary earners represented by salary earners. Data are from *Fifteenth Census of the United States: 1930*, "Mines and Quarries: 1929," U. S. Dept. Com., Bur. Census, 1933, "General Report," p. 9. Data include employees in central administrative offices.

[21] The index of production for manufactured goods rose from 28.1 in 1899 to 43.4 in 1909 compared with an increase in the number of salary earners in the corresponding period from 364,000 to 790,000. Between 1919 and 1929, however, the production index rose from 63.4 to 100.0 while the number of salary earners increased from 1,438,000 to 1,567,000. Index of production computed by splicing data for 1899–1919 from F. C. Mills, *Economic Tendencies in the United States*, New York, National Bureau of Economic Research, 1932, pp. 92 and 192, to data for 1919–29 obtained from *Production, Employment and Productivity in 59 Manufacturing Industries*, WPA National Research Project, May, 1939, Part 1, p. 65. For source of salary-earner data see footnote 19 above.

For one thing, to a far larger extent than consumers' commodities the consumers' services have the character of luxuries, at least in the sense that they do not supply the necessaries of life and can either be postponed or performed by the consumers themselves, or forgone altogether. Second, practically all consumers' commodities are manufactured for sale at a profit. In anticipation of profits, savings are invested and labor is employed; to maximize profits, the technology is continually being improved either to permit price reductions and increased sales or to widen the profit margin. On the other hand, a large part of the labor used to perform personal services is not employed for profit. This is especially true of the domestic servants employed in private families. Since the person who pays for the service of a servant does not derive a monetary return for such employment, there is no basis for the expansion of such employment except from an increase in the flow of income from other pursuits. The character of the domestic and personal services performed and the conditions under which they are usually performed are moreover such that there is little room for the development of a machine technology. However, as more and more of such services are taken out of the sphere of the domestic economy and become commercialized, they also increasingly take on the characteristics of the consumers'-goods industries in the sense that they offer, on the one hand, opportunities for investment and, on the other hand, opportunities for technological improvements.

The Census considers these service activities under two groups—Domestic and Personal Service, and Professional Service, Recreation, and Amusement. The first of these, domestic and personal service, is by far the larger. (See Table 6.) This is also the group which in recent years has grown less rapidly than the population. From 1870 to 1910 the population increased 131 per cent, while the number of persons attached to domestic and personal service tripled. From 1910 to 1930, however, the population increased 33 per cent and the number of persons attached to domestic and personal service rose only 28 per cent. (See Table 1.) This change in the rate of growth reflects primarily the slow rate of increase of persons attached to domestic service in private families and in noncommercialized services. For example, there was an absolute decline in the number of launderers and laundresses in private families between 1910 and 1930, while the largest increases during that period occurred in the number of persons attached to commercial laundries and cleaning and dyeing and pressing shops. The other groups which showed large increases were persons attached to apartment house and building maintenance, barbers, hairdressers, and manicurists, and persons attached to hotels, restaurants, and boarding houses.

As may be seen from Table 6, the commercialized services to persons increased from 1910 to 1930 at a faster rate than the population. Esti-

mates of consumers' expenditures for intangibles, broken down according to personal and small-shop services and larger-scale commercial services, show that from the "prewar average" to 1919 consumers' expenditures (in current dollars) for personal and small-shop services

TABLE 6

NUMBER OF PERSONS ATTACHED TO DOMESTIC AND PERSONAL SERVICE, PROFESSIONAL SERVICE, AND RECREATION AND AMUSEMENT, 1910, 1920 AND 1930[a]

(In thousands)

	1910	1920	1930
Domestic and personal service	3,771	3,393	4,815
Hotels, restaurants and boarding houses	948	913	1,357
Launderers and laundresses (not in laundry)	534	397	361
Laundries, cleaning and dyeing and pressing shops	174	194	420
Barbers, hairdressers, and manicurists	195	216	374
Apartment house and building maintenance[b]	131	178	298
Domestic service in private families[c]	1,491	1,299	1,966
Other domestic and personal service	298	195	38
Professional service, recreation, and amusements[d]	1,114	1,496	2,246
Art, recreation, and amusement[e]	282	295	435
Education[f]	77	110	174
Health[g]	302	403	618
Clergymen and religious workers	126	148	180
Other (accounted for)			
Legal profession[h]	105	109	139
Supplying services to industry[j]	108	128	139
Miscellaneous[k]	26	42	61
Not distributed[d]	87	260	501

[a] Computed from Daniel Carson assisted by Henrietta Liebman, *Labor Supply and Employment, Preliminary Statement of Estimates Prepared and Methods Used*, WPA National Research Project, mimeo., November, 1939. Based on *Fifteenth Census of the United States: 1930*, "Population," U. S. Dept. Com., Bur. Census, 1933, Vol. V, Ch. 2 and Ch. 7, and *Thirteenth Census of the United States: 1910*, "Population," U. S. Dept. Com., Bur. Census, 1914, Vol. IV, Table 6. All series are comparable to the industrial classification of the 1930 Census, as adjusted and reclassified in the above-mentioned National Research Project report.

[b] Estimated. Includes charwomen and cleaners, elevator tenders, janitors and sextons, etc., assignable to apartment-house and building maintenance.

[c] Includes those housekeepers, stewards, cooks, and servants (not in hotels), untrained nurses, chauffeurs and coachmen, and waiters who are assignable to domestic service in private families.

[d] It was not possible to distribute the "repeater" occupations (viz., laborers, janitors, clerical assistants, etc.) to each of the subdivisions so that the subgroups do not represent complete industrial classification. Each subdivision includes only characteristic occupations assignable to professional service (on the basis of the 1910 and 1930 industrial distribution). The "repeater" occupations are included in the "not distributed" classification. It should be noted that inasmuch as these data are classified on an industrial basis, they exclude those members of the several professions who are directly attached to other industries. For example, doctors employed by a manufacturing firm will be included under manufacturing. For a purely occupational breakdown for selected occupations see Table 7.

[e] Includes the characteristic occupations in recreation and amusement, and artists, sculptors and teachers of art, teachers (athletic, dancing, etc.,) and authors.

[f] Includes only private-school teachers and college professors and instructors. Public-school teachers have been shifted to public service.

[g] Includes dentists, physicians and surgeons, veterinary surgeons, trained nurses, physicians', surgeons', and dentists' attendants, healers, and social and welfare workers.

[h] Includes lawyers, abstractors, notaries, and justices of the peace.

[j] Includes architects, chemists, assayers and metallurgists, civil engineers and surveyors, etc.

[k] Includes keepers of charitable and penal institutions, officials of lodges and societies, librarians, librarians' assistants and attendants.

Note: All figures were rounded after computations were made.

increased about 50 per cent, while the expenditures for larger-scale commercial service increased over 80 per cent; from 1919 to 1929 the expenditures for the former increased about 50 per cent, while those for the latter increased about 160 per cent.[22]

These changes from domestic and small-shop to larger-scale commercial services may reflect a trend in the consumption pattern of the community, but they are of interest from a number of other angles.

[22] William H. Lough, *High-Level Consumption*, New York: McGraw-Hill Book Co., Inc., 1935), p. 44.

To the extent that the shift was from a domestic to a commercial basis, it represents a shift into the market and profit economy of a service which was formerly largely outside that economy. Moreover, the large-scale commercial services require more capital, and in some respects utilize a machine technology, as in laundries and dry-cleaning establishments, which increases the productivity of labor and thereby reduces the labor required for a given quantity of services. The growth in large-scale commercial services also represents a shift from self-employed small shopkeepers to wage and salary employment and therefore to a status which, by and large, is more precarious so far as attachment to a source of income is concerned. It has been pointed out that the expenditures for personal and small-shop services stood up reasonably well under the impact of the depression of the early 1930's whereas the larger-scale enterprises which "shot up at high speed to their 1929 peak . . . dropped their gains with even greater celerity and no effective resistance in the slump of 1929–1931."[23]

The second subgroup, the professional, recreational and amusement services, which is shown in Table 6, grew much more rapidly than the domestic and personal services.[24]

The statistics on persons attached to the health services (see Tables 6 and 7) show that although these services as a group grew very rapidly between 1910 and 1930, the number of physicians and surgeons remained practically constant and therefore actually declined on a per capita basis. Only between 1870 and 1880 and between 1890 and 1900 did the number of persons attached to these occupations increase per capita, and even then the increase was slight. Dentists and dentists' assistants, on the other hand, showed significant increases both in absolutes and in per capita, but even their per-capita increase between 1920 and 1930 was much smaller than in the previous decade. The significant increases occurred in the number of trained nurses, which tripled between 1910 and 1930. These differences in the trends of the occupations which compose this group of services undoubtedly reflect the steady growth of hospitals and clinics.

[23] *Ibid.*, p. 43.

[24] In the analysis of the data on professional services we must keep in mind a number of qualifications. The Census industrial classification of the professionals includes (in addition to Education, Health, Religion, Recreation and Amusement, and the employees of nonprofit-making institutions) those technical and legal professionals who, though they supply in the main services to industry, are not employed in industrial establishments. They are either self-employed, or employed in establishments that supply primarily these technical and legal professional services. Since it was not possible to segregate the clerical and manual assistants employed in establishments supplying professional services to industry, trade, and finance, they had to be included here.

Clergymen and religious workers have been increasing per capita during the past 60 years, but their increase since 1910 has been relatively slow. On the other hand, the group of occupations classified by the Census under "recreation and amusement" remained practically stationary from 1910 to 1920 but experienced a considerable increase from 1920 to 1930.[25] The increase between 1920 and 1930 reflects to a

TABLE 7

NUMBER OF PERSONS ATTACHED TO SELECTED OCCUPATIONS IN PROFESSIONAL SERVICE AND AMUSEMENT, DECENNIALLY, 1870–1930[a]

(In thousands)

	1870	1880	1890	1900	1910	1920	1930
Population[b]	39,818	50,156	62,622	75,995	91,972	105,711	122,775
Selected occupations:							
Physicians and surgeons[c]	62	86	105	132	158	165	189
Dentists	8	12	17	30	40	56	71
Trained nurses	5[d]	6[d]	18[d]	46[d]	82	149	294
Clergymen and religious workers[e]	44	65	88	112	134	168	211
Artists, sculptors and teachers of art, and authors	4	10	29	31	38	42	70

[a] These groups are distributed by occupation and include the total in all industries of those attached to a given occupation. Data for: 1870–1900 are from the *Twelfth Census of the United States: 1900*, "Special Reports," U. S. Dept. Com. and Lab., Bur. Census, 1904, "Occupations," pp. xxxii–xlix; 1910–1930 from *Fifteenth Census of the United States: 1930*, "Population," U. S. Dept. Com., Bur Census, 1933, Vol. V, Ch. 2.

[b] Table 1.

[c] Includes healers, chiropractors, and osteopaths.

[d] The number of nurses in these years was estimated by multiplying "total nurses and midwives" in each year by the 1910 ratio of trained nurses to nurses and midwives.

[e] Includes social and welfare workers. It was desirable to classify this group under "health"; since, however, segregation is possible only for 1910, 1920, and 1930, social and welfare workers were carried in this table for purposes of comparability with earlier years in combination with clergymen and religious workers. In Table 6 this group was classified under "health" (see footnote g to that table).

Note: All figures were rounded after computations were made.

large extent the growth of the moving-picture industry, but it fails to reflect the growth in recreation and amusement activities and employment that took such forms as recreational travel, resort activities, and sport activities.[26]

(5) We come now to the services performed by public institutions. Public service activities and employment in public service have grown at a more rapid rate than the population.[27] The increasing functions of

[25] This is a rather heterogeneous group and includes artists of all kinds, teachers of art, authors, actors, and such other occupations connected with the theater as stage hands and ushers, and persons in occupations attached to race tracks, dance halls, billiard rooms, etc. The statistics on these occupations are poorly adapted for a measure of what happened to employment opportunities in the field of recreation and amusement.

[26] Mr. Lough's analysis (see *op. cit.*, *supra*) of different types of "consumer takings" shows that recreation was one of the few categories which increased considerably during the twenties.

[27] From 1870 to 1910 total population increased by 130 per cent while the number of persons attached to public service increased by 450 per cent. Between 1910 and 1930 the 33-per-cent increase in population was accompanied by a 100-per-cent increase in public service. (See Table 1.)

government connected with the growth and complexity of business, the growth in urbanization, and the increase in postal service, public education, and national defense activities have all contributed to this phenomenon. But principally the increases in public-service employment reflect the widening of the field of public activity in other directions. Increasingly government has assumed the functions of education, water supply, road construction, sanitation, public health, irrigation, land reclamation, and soil conservation, and it has vastly enlarged the scope of social legislation and its administration.

The Census of Occupations data are neither detailed enough nor comprehensive enough to permit an analysis of the trends of employment in all types of public service. There are, however, some data on the major categories.

Public education is the largest of the governmental functions both in employment and in expenditures. The trend in educational activities in the United States has been steadily upward. This trend reflects not only population growth and the popular pressure for free public education, but also the needs of industry for an increasing number of clerical employees, and for a greater schooling of its labor forces to man the increasingly complex industrial machinery.

From 1870 to 1900 the rate of increase in public-school enrollment was greater than that of the number of children of school age and greater than the rate of increase in public-school teachers. During the first three decades of this century, however, the growth in the number of teachers had outstripped even pupil enrollment. One reason was the increase in the percentage of enrolled children attending school each day and in the average number of days that the schools were in session. Another reason was the growth of secondary-school education. In 1890, 95 per cent of the population 5–13 years of age were enrolled in elementary public schools, but only 4 per cent of the population 14–17 years of age were enrolled in secondary public schools. By 1930 this percentage had risen to 47.[28] In more recent years (from 1930 to 1936), as a reflection of the changing age composition of the population, the number of elementary public-school teachers declined, while the number of secondary public-school teachers continued to increase.[29]

[28] Data computed from issues of the *Biennial Survey of Education in the United States*, "Statistical Summary of Education," U. S. Dept. of Interior, Office of Education.

[29] ". . . the absolute number of children of elementary school age has already begun to decline. By 1940 there will be about 5 per cent fewer children 6–9 years of age than in 1920. The total number aged 6–13 will be about equal to that in 1920 and 1½ million less than in 1930. The decrease in elementary-school enrollment which became noticeable in 1930 will thus continue. The number of children of high-school age (14–17 years) will continue to increase until about 1940, after

To complete the picture for education it should be noted that about 10 per cent of all elementary- and secondary-school teachers are engaged by private schools. In general, since 1870 the proportion of pupils enrolled in private elementary schools to those enrolled in public elementary schools has increased slightly, and the number of teachers employed in private elementary schools has run parallel to enrollment. There has, however, been a marked decline in the proportion of teachers employed in private secondary schools as compared with public secondary schools.[30]

TABLE 8

NUMBER OF PERSONS ATTACHED TO PUBLIC SERVICE, 1910, 1920, AND 1930[a]
(In thousands)

	1910	1920	1930
Public service	1,244	1,882	2,490
Federal, state, county and city government	457	672	906
National defense	83	244	143
Postal service	170	209	284
Public school education	534	757	1,157

[a] Total public service as shown in this table includes the Census group "Public service (not elsewhere classified)," and in addition postal service and public-school education. The phrase "not elsewhere classified" in the title of the Census group means of course that an all-inclusive total would have to embrace all those engaged in public service who are elsewhere classified. Such a comprehensive total cannot, however, be constructed on the basis of presently available Census data. See also footnote 6.

Data were computed from Daniel Carson assisted by Henrietta Liebman, *Labor Supply and Employment, Preliminary Statement of Estimates Prepared and Methods Used*, WPA National Research Project, mimeo., Nov. 1939, based on *Fifteenth Census of the United States: 1930*, "Population," U. S. Dept. Com., Bur. Census, 1933, Vol. V, Ch. 2 and Ch. 7 and the *Thirteenth Census of the United States: 1910*, "Population," U. S. Dept. Com., Bur. Census, 1914, Vol. IV, Table 6. All series are comparable to the industrial classification of the 1930 Census as adjusted and reclassified in the above-mentioned report.

Note: All figures were rounded after computations were made.

Of the remaining classifiable public services, the postal service grew from 170,000 in 1910 to 284,000 in 1930. National defense activities increased from 83,000 to 143,000 in the same period; and city, state, and federal government functions absorbed 457,000 in 1910 as compared with 906,000 in 1930. (See Table 8.)

For additional light on governmental activities we have to fall back on expenditure figures and even here our scope is restricted to the period since 1915. Analyzing governmental expenditures for the years 1915 to 1930, Carroll H. Wooddy found that highway construction and maintenance, education, and public-welfare activities together with national defense activities accounted for three-fifths of the governmental

which the downward trend will begin for that age group as well." *The Problems of a Changing Population*, National Resources Committee, Washington, 1938, p. 195.

[30] There were about three and a half times as many teachers in colleges in 1930 as in 1900. Enrollment in colleges increased about four-and-a-half-fold. In 1900 about 4 per cent of the population 18–21 years of age was enrolled in colleges; by 1930 about 12 per cent. Data are computed from issues of the *Biennial Survey of Education in the United States* (*supra*).

expenditures in 1915 and had grown to two-thirds of the total by 1930.[31]

In other branches of governmental activity, the most impressive development in the civil branch of the federal service was the marked expansion of activities concerned with commerce, industry, transportation, and communication. Part of this growth arose from the extension of regulatory activities, particularly those relating to interstate trade, railroads, and shipping. A much greater part was due to the development of services to business in the form of aids to foreign and domestic commerce, to agricultural marketing, and to commercial aviation; the rapid growth of the postal service and the attempt to build up a domestic merchant marine were special features of this trend. The conservation and development of natural resources, including the provision of aids to agriculture, increased both in federal and state governments at about the same rate as civil activities as a whole. Public-health activities diminished slightly in relation to other functions of federal and state governments while cities and other local governments were assuming an increasing burden in this field.

As to the question whether the growing governmental expenditures represented new activities or the expansion of old activities, Wooddy estimated that while about one-half of the growth of federal expenditures can be ascribed to new types of functions, on the state and local levels the increases up to 1930 had largely occurred in the fields of education and highway construction and maintenance, that is, through the expansion of old activities.[32] The results of the 1940 census are of course likely to show a different picture and will reflect the effects of the social legislation of the 1930's with the consequent increase in employment in public-welfare activities.

III

In 1930 the activities of roughly 45 per cent of the total persons attached to the so-called service industries were classifiable as being concerned mainly with transportation, distribution, or other types of handling of commodities. Roughly another 7 per cent consisted of persons who were attached to the financial and property-handling institutions, that is, to activities that are also concerned with functions which are related to the production and distribution of goods. If we add the salaried employees who are directly attached to the commodity-producing industries, we find that the employment prospects of well over

[31] Carroll H. Wooddy, "The Growth of Governmental Functions," in *Recent Social Trends in the United States*, Report of the President's Research Committee on Social Trends; New York, McGraw-Hill Book Co., 1933, Chapter XXV, pp. 1325–26.

[32] *Ibid.*, p. 1328.

half of the people who look to the service industries for a livelihood are directly linked to the economic fortunes of the commodity-producing industries.

These service industries and occupations can expect an increase in employment opportunities only if the output of commodities should increase, and even then the extent of the increase in jobs will be influenced by advances in the technology with which the services are performed. In the industries that are based on a machine technology with continuing and rapid increases in productivity, employment is likely to grow, if at all, much less rapidly than the volume of production. The railroads are a case in point. On the other hand, in those industries that, like the distributive trades, represent primarily services performed by persons, the opportunities for productivity increases are much smaller and their employment trends are therefore likely to stand in much more direct relationship to the volume of goods and services produced.

The domestic, personal, professional, and recreational services embraced in 1930 about 7 million persons, or about one-third of those attached to the services. To the extent that the shifts from a domestic to a commercial basis are likely to continue in the future and to the extent that the commercialized services utilize modern production techniques, the opportunities for reducing the amount of labor required for a given quantity of services are likely to multiply in the future. Even the maintenance of and certainly an increase of employment in such service activities will require a rapid expansion of these services. Such an expansion, especially of those commercialized services which utilize a modern machine technology, would also contribute to expansion in the rest of the economy through the investment demand which they would create and would thus increase the demand for their own services. But the demand for these services as a whole depends on the volume and distribution of the personal incomes which are derived from other economic activities.

An increase in the flow of income to those individuals and families who now use little or none of the personal services should increase the demand for such services. The minimum-wage and maximum-hour laws, unemployment and old-age insurance, the spread of collective bargaining, the maintenance of public-work programs, and similar social measures, by raising the income of the economically lowest strata of the population, may indeed contribute to an increasing demand for personal service as well as for consumers' goods. However, a constantly expanding purchasing power to exert an increasing demand for these goods and services depends on a continually increasing total national income available for distribution. Since a steadily decreasing amount of labor is required to produce a given volume of goods, the mainte-

nance and expansion of purchasing power depend ever more on the expansion of investment in producers' durable goods, either for the expansion of old industries or the building of new ones. Whenever such investments slow down, the volume of labor employed slows down, the production of goods slows down and with them the services that transport and sell these goods, that manage the resources required for their production, and that depend for their continuation on consumers' incomes derived from their production. Thus the transportation and commodity-handling services, the financial and property-handling services, the salaried employees of the commodity-producing industries, and the domestic and personal services, that is, the services which collectively include from eight- to nine-tenths of the persons attached to all service industries, can hardly be expected to experience an expansion without an expansion of the capital-goods producing industries. And it is these capital-goods industries which have notably lagged behind during the comparative recoveries since 1932–33. Whatever will serve as a stimulus to their expansion in the future will also enhance the employment prospects of the services.

Although the public services, too, are ultimately dependent on the incomes which the rest of the population derives in the process of producing other goods and services, at any one time neither the criteria for the expansion of public employment nor the source of the funds required for it need stand in direct relationship to business conditions. For the need for governmental action and the pressure to borrow or tax may change in reverse order to the functioning of private business as an employer of labor and as a source of current income. Those public services included in 1930 more than one-eighth of the total number of persons attached to the services and their relative importance has doubtless increased during the thirties. The importance of governmental action in the economic life of the country is, however, not measurable in terms of the number of people employed by governments. Rather, its importance has to be gauged in terms of the contributions which governments make to production and to national income. Although the size of that contribution is a matter of controversy, there is little doubt that federal, state, and local governmental expenditures for all types of construction have in the past been an important contributing factor in developing opportunities for private investment. If private investment continues to lag in the future, the stimulus required for the expansion of the capital-goods industries, and through them of the other industries and services, may have to come through an expansion of capital outlays on the part of governments.

WPA National Research Project

THE INADEQUACY OF TESTING DYNAMIC THEORY BY COMPARING THEORETICAL SOLUTIONS AND OBSERVED CYCLES*

By TRYGVE HAAVELMO

IN MODERN business-cycle research the following proceeding is being commonly used: First, a mathematical model (a determinate dynamic system) is set up, as an attempt to describe approximately the interconnections between a set of economic variables, their time derivatives, lagged values, and so on, in terms of strict functional relations. By some statistical procedure the constants in the system are estimated from the corresponding observed time series. Then the system is "solved," i.e., the variables are expressed as explicit functions of time, involving the estimated parameters. The degree of *conformity* between these theoretical solutions and the corresponding observed time series is used as a test of the validity of the model. In particular, since most economic time series show cyclical movements, one is led to consider only mathematical models the solutions of which are cycles corresponding approximately to those appearing in the data.[1] This means that one restricts the class of admissible hypotheses by inspecting the *apparent form* of the observed time series.

This condition for a "good" theory is of course not a sufficient one, since there are in general many *different* a priori setups of theory which are capable of reproducing approximately the observed cycles. But, what is more important, it may not even be a necessary condition, and its application may result in a dangerous and misleading discrimination between theories. The whole question is connected with the *type of errors* we have to introduce as a bridge between pure theory and actual observations. Compared with actual observations, each equation in a dynamic model splits the observed variations into two parts, one part which is "explained" by the equation, and another part which is not accounted for, and which is ascribed to external factors. This kind of splitting is common to all theory. We usually consider such equations as "good" and "useful" theories if, in order to get full agreement between theory and observations, it *is and continues to be* sufficient to allow for only relatively small and random external factors.

There are two main ways in which such external factors may be

* The author feels much indebted to Dr. J. Marschak and Dr. A. Wald for reading the manuscript and for many valuable comments.

[1] E.g., M. Kalecki, ("A Macrodynamic Theory of Business Cycles," ECONOMETRICA, Vol. 3, No. 3, July, 1935, p. 336), goes so far as to impose on his system not only the condition of cyclical solutions, but also the condition of constant amplitude (no damping) in order to produce maintained oscillations which can be directly compared with the observed cycles.

combined with the theoretical setup, and they have fundamentally different consequences for our "explanation" of observed cycles.

One way is to consider the set of fundamental equations as exact equations without errors, i.e., they describe an ideal hypothetical model. We may perform certain elimination operations in order to get a set of *final* equations, each containing only one unknown time function. Solving these final equations we obtain the general theoretical form of the time expansions of each variable. Then we may adjust the parameters at our disposal in these general solutions so as to make the theoretical solutions fit the observed time series "as well as possible."[2] Following this approach we are led to consider the difference between theoretical and observed series as additive errors *superimposed* on the theoretical time movement of the variables. (This assumption underlies the method of periodogram analysis and other mechanical "decomposing" methods.) If these superimposed errors are supposed to be random, and if the observed series show clear cycles, there must be cycles in the theoretical ("error-free") time expansions. If not, we may be justified in rejecting the theory as unrealistic.

Another way is to introduce the errors explicitly in the original set of fundamental equations describing our model. Then we have to carry these errors along in the elimination process, and we end up with final equations which contain certain stochastical elements. This latter scheme turns out to be the one actually chosen in most of the modern studies of dynamic systems.[3] Usually this is not explicitly stated; but it is implied in the commonly used proceeding of fitting the fundamental equations in a dynamic system to data in order to get estimates of the coefficients. All such fittings admit some unexplained residuals—in the best case—of a random character. Such errors may not seriously affect the "explanatory" value of each fundamental equation taken separately, i.e., we may get a close connection between "calculated" and "observed" values for each of these equations. Here the unexplained residuals enter merely as errors of estimation, and they may be small. But the same does not apply to the *form* of the solution we obtain from the final stochastical equations. Here the

[2] See, e.g., F. W. Dresch, "A Simplified Economic System with Dynamic Elements," Cowles Commission for Research in Economics, *Report of Fifth Annual Research Conference . . .*, 1939, pp. 18–21. He writes (p. 20): "In such a solution each variable of the system will be expressed as a function of time in terms of the constants defining the assumed functions. By choosing the values of these constants in such a way that the theoretical time curves for these variables correspond as well as possible (in some sense or other) to the observed time series for these variables, one can "fit" such a model to the actual economy."

[3] See, in particular, several recent works of Professor Tinbergen and his followers.

error terms play a much more fundamental role. Indeed, the form of the final solutions (the time series) obtained if errors are neglected throughout and the form obtained when the errors are taken account of may show widely different patterns. From the fact that each fundamental equation taken separately shows only small random errors (i.e., our model is realistic) there does not necessarily follow any close similarity between the *form* of the observed series and that of the theoretical solutions obtained by neglecting the errors. In particular, the solutions obtained by neglecting all error terms may not show cycles at all, while clear cycles appear as soon as the error terms are included. Therefore, if we admit certain error terms in the system of fundamental equations, we have to investigate the effect of these errors upon the shape of the final solutions. It must be noticed that the assumption of errors in the system of fundamental equations is—except in very particular cases—incompatible with an assumption of simple superimposed errors in the final solutions as described above.

We shall now consider a constructed example which will throw some light upon these questions. We shall choose a very simple scheme, frequently occurring in economic dynamics.

Let $x(t)$ be an observable economic time series, and let the elimination result of a dynamic system be

$$(1)\quad x(t) + a_1x(t-1) + a_2x(t-2) = \epsilon(t-1) \qquad (t = t_0 + 2, t_0 + 3, \cdots),$$

where a_1 and a_2 are constants and $\epsilon(t)$ a random variable with expectation equal to zero and constant finite variance for all integral values of t and zero elsewhere; t_0 is the initial point of time. This means that $x(t)$ is not uniquely determined by the past, because each "year" new things happen. Such assumptions, in one form or another, underlie all dynamic theories which claim to have some relevance to facts. We write $\epsilon(t-1)$ instead of $\epsilon(t)$ because it is more realistic to assume that the external factors do not have immediate consequence for the variable considered. Moreover, it simplifies our example as shown below.

Now let us first consider the solution of (1). The principal solution of the *homogeneous* equation

$$(2)\qquad x(t) + a_1x(t-1) + a_2x(t-2) = 0$$

is

$$(3)\qquad x_1(t) = Ae^{\rho_1(t-t_0)} + Be^{\rho_2(t-t_0)},$$

where

$$(4)\qquad e^{\rho_1} = \frac{-a_1 + \sqrt{a_1^2 - 4a_2}}{2}, \qquad e^{\rho_2} = \frac{-a_1 - \sqrt{a_1^2 - 4a_2}}{2},$$

ρ_1 and ρ_2 being real or complex numbers, and A and B arbitrary constants. Now let

$$\lambda(\tau) \qquad (\tau \geqq 0) \tag{5}$$

be a particular integral of (2) obtained from (3) by assigning to A and B certain specific values to be disposed of later. $\lambda(\tau)$ therefore satisfies the difference equation

$$\lambda(\tau+2) + a_1\lambda(\tau+1) + a_2\lambda(\tau) = 0 \qquad (\tau \geqq 0). \tag{6}$$

Consider the expression:

$$v(t) = \sum_{0}^{t-t_0}{}_{\tau}\lambda(\tau)\cdot\epsilon(t-\tau). \tag{7}$$

We may write down the identity

$$\begin{aligned} v(t) + a_1v(t-1) + a_2v(t-2) &\equiv \sum_{0}^{t-t_0}{}_{\tau}\lambda(\tau)\epsilon(t-\tau) \\ &+ a_1 \sum_{0}^{t-t_0-1}{}_{\tau}\lambda(\tau)\epsilon(t-\tau-1) + a_2 \sum_{0}^{t-t_0-2}{}_{\tau}\lambda(\tau)\epsilon(t-\tau-2). \end{aligned} \tag{8}$$

The right-hand side of (8) may be written

$$\begin{aligned} &\sum_{0}^{t-t_0-2}{}_{\tau}\lambda(\tau+2)\epsilon(t-\tau-2) + a_1 \sum_{0}^{t-t_0-2}{}_{\tau}\lambda(\tau+1)\epsilon(t-\tau-2) \\ &\qquad + a_2 \sum_{0}^{t-t_0-2}{}_{\tau}\lambda(\tau)\epsilon(t-\tau-2) \\ &\qquad + \lambda(0)\epsilon(t) + \lambda(1)\epsilon(t-1) + a_1\lambda(0)\epsilon(t-1) \\ &\equiv \sum_{0}^{t-t_0-2}{}_{\tau}\{\epsilon(t-\tau-2)[\lambda(\tau+2) + a_1\lambda(\tau+1) + a_2\lambda(\tau)]\} \\ &\qquad + \lambda(0)\epsilon(t) + [a_1\lambda(0) + \lambda(1)]\epsilon(t-1), \end{aligned}$$

where the first expression in square brackets $=0$ because of (6). Hence

$$v(t) + a_1v(t-1) + a_2v(t-2) = \lambda(0)\epsilon(t) + [a_1\lambda(0) + \lambda(1)]\epsilon(t-1). \tag{9}$$

Now we shall dispose of the arbitrary constants in $\lambda(\tau)$ so that

$$\begin{aligned} \lambda(0) &= 0, \\ \lambda(1) &= 1. \end{aligned} \tag{10}$$

Then $v(t)$ satisfies a difference equation which is identical with (1). The complete solution of (1) is therefore, by adding (3) and (7),

$$x(t) = x_1(t) + \sum_{0}^{t-t_0}{}_{\tau}\lambda(\tau)\epsilon(t-\tau). \tag{11}$$

$\lambda(\tau)$ is here completely determined by (6) and (10). It is seen that, if (3) represents a damped sine curve or two damped exponentials, then $x_1(t)$ will practically vanish after some time and then $x(t)$ depends almost entirely upon the cumulation process $v(t)$. Now $v(t)$, being a linear combination of ϵ's, is a random variable with expectation equal to zero for each point of time taken separately. But $v(t)$ and $v(t-1)$ will in general be serially correlated, the correlation depending upon the weights $\lambda(\tau)$. When the weights $\lambda(\tau)$ are damped the variances $\sigma^2_{v(t)}$ of $v(t)$ will be finite for every t, and for large $(t-t_0)$ they approach the upper limit

$$\sigma_v^2 = \sigma_\epsilon^2 \cdot \sum_{0}^{\infty}{}_{\tau}\, \lambda^2(\tau), \tag{12}$$

where the sum is certainly convergent when $\lambda(\tau)$ is damped exponentially. The smaller the damping the greater will be the average amplitude of $x(t)$. It is well known that, when $\lambda(\tau)$ is a damped sine curve, the series $v(t)$ will show distinct cyclical movements with a principal period corresponding on the average to that of the harmonic factor in $\lambda(\tau)$. But also when $\lambda(\tau)$ is a sum of pure damped exponentials there will in general be some cyclical movements in $v(t)$. Intuitively this may be seen by the following simple considerations: When $\lambda(\tau)$ is damped, $v(t-\kappa)$ and $v(t)$ will, for sufficiently large κ, be practically independent in the probability sense. Therefore, as $v(t)$ has expectation zero, the average of $v(t)$ over a long period must tend to zero as the variance of $v(t)$ is finite. But because $v(t)$ is certainly not always zero, as is seen from (12), $v(t)$ will have to oscillate in some way around zero, and because of the serial correlation in $v(t)$ these oscillations will show some "stickiness." For example, positive serial correlation between $v(t)$ and $v(t-1)$ increases the probability of iterations, i.e., consecutive positive or consecutive negative terms in $v(t)$ as compared with $\epsilon(t)$.[4]

We shall discuss a constructed example corresponding to scheme (1) where the solution of the homogeneous equation (2) is a sum of two damped exponentials. The example is

$$x(t) - 1.2x(t-1) + 0.3x(t-2) = \epsilon(t-1). \tag{13}$$

The random series $\epsilon(t)$ was constructed from results of drawings in the Danish Class Lottery. Each $\epsilon(t)$ is the average of 10 independent ob-

[4] Methods for determining a priori the principal characteristics of such cumulative cycles have been worked out by Professor Frisch at the Institute of Economics, Oslo, but they have not yet been published.

servations from a rectangular distribution of the integers 0, 1, 2, $\cdots$, 9. The expected value, 4.5, was subtracted; therefore

$$E(\epsilon) = 0, \tag{14}$$

$$\sigma_\epsilon^2 = 8.25. \tag{15}$$

The series $\epsilon(t)$ is shown in Figure 2.

The homogeneous equation

$$x(t) - 1.2x(t-1) + 0.3x(t-2) = 0 \tag{16}$$

has the solution

$$x_1(t) = Ae^{-0.17t} + Be^{-1.04t}, \tag{17}$$

where A and B are arbitrary constants. Choosing $\lambda(0)=0$, $\lambda(1)=1$, the weights $\lambda(\tau)$ become

$$\lambda(\tau) = 2.04(e^{-0.17\tau} - e^{-1.04\tau}), \tag{18}$$

the graph of which is shown in Figure 1. This curve shows (apart from a constant factor) the time development of $x(t)$ which would fol-

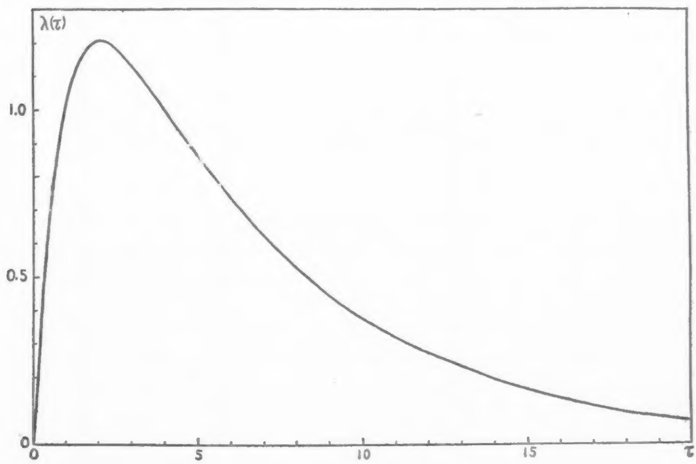

FIGURE 1.—The structural movement of the system [formula (18)].

low if $x(t)$ had been zero for two or more years, then suddenly received a positive impulse, and later were allowed to move uninterruptedly.

Now there are hardly any economic series the movements of which show resemblance to those of Figure 1: "The theory does not describe

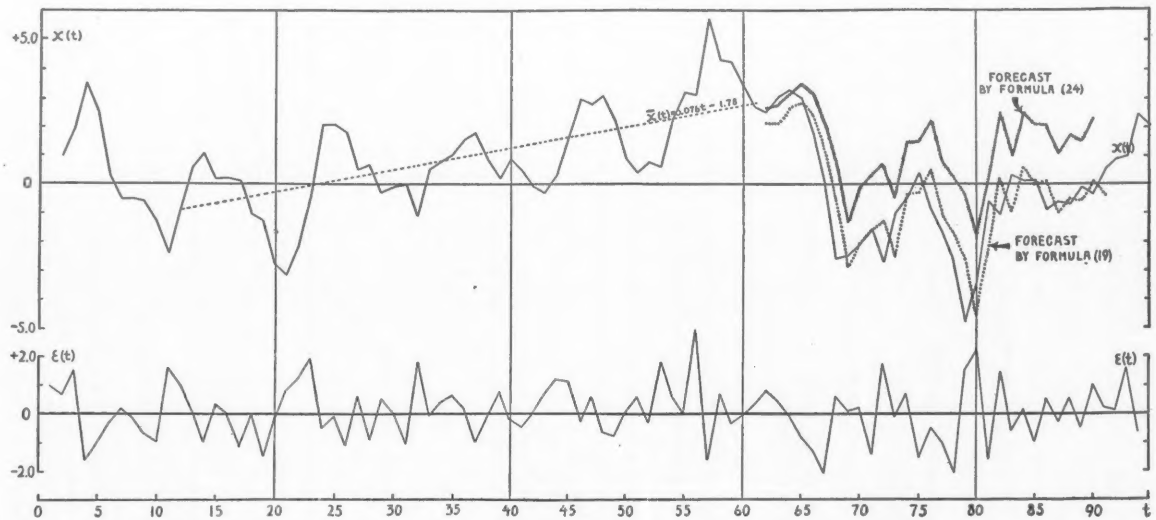

FIGURE 2.—Constructed example of shock cumulation [see formula (13)]. Lower part of figure shows the purely random series $\epsilon(t)$.

the facts." But here we must distinguish between two different ways of using the theory (16). When we want to calculate $x(t)$ in terms of the *previously observed* values $x(t-1)$ and $x(t-2)$, $\epsilon(t)$ plays only the role of errors of estimation, and if the ϵ's are small (16) is a "good" theory. But when we want to study the observed *form* of $x(t)$ we must consider the general equation (13). In Figure 2 the series $x(t)$ has been calculated step by step by means of (13), starting from two arbitrary initial conditions chosen close to zero. When we speak of cycles in economic data, we hardly think of more regular waves than the 9–10 "years" cycle shown by this curve. The series has a longer wave too, and none of these characteristic movements appear in the series λ or ϵ. The argument that a theory giving noncyclical theoretical solutions should be rejected when the observations show cycles, therefore, cannot be assumed to have general validity.[5]

Let us assume that the theory (2) has been well established on careful a priori considerations. And suppose, for example, that our observation material is the curve $x(t)$ in Figure 2 for the 52 years from $t=10$ to $t=61$, inclusive, the later observations being in the future. This part of the curve has very distinct cycles, and it also seems to have an approximately linear "trend." This "trend" looks unfavorable to our a priori theory. Nevertheless, having the general equation (1) in mind, and fitting the equation (2) to the observations, we shall obtain a useful result. Indeed, taking the regression of $x(t)$ on $x(t-1)$ and $x(t-2)$, which gives a consistent estimate of the coefficients, since $x(t-1)$ and $x(t-2)$ do not depend upon $\epsilon(t-1)$, we obtain

$$x(t) - 1.1x(t-1) + 0.26x(t-2) = 0. \tag{19}$$

A period covering 52 observations ($t=10$ to $t=61$) was used in order to have 50 observations net of each variable in (19). The solution of (19) gives two damped exponentials, $e^{-0.29t}$ and $e^{-1.05t}$, showing a little heavier damping than the theoretical. Taking the deviations between observed and calculated values of $x(t)$, we should see that they are random, and having the general equation (1) in mind, we should conclude that it is sufficient to allow for some random events in order to have full agreement between our theory and the observations. The fitting to data has added to our knowledge the fact that the structural movements of the system are exponentials, not cycles. This question was left open in the a priori theory. The theory was also *capable* of giving cycles.

[5] For actual examples from economic data where the theoretical solutions turned out to be pure damped exponentials, see the author's article "The Method of Supplementary Confluent Relations, Illustrated by a Study of Stock Prices," ECONOMETRICA Vol. 6, No. 3, July, 1938, pp. 216–218.

The usefulness of (19) for prediction purpose is shown in the dotted curve ($t=62, 63, \cdots$) in Figure 2, where $x(t)$ is forecasted *one year ahead* by means of (19), using observed values of $x(t-1)$ and $x(t-2)$. It is seen that the "best" forecast comes almost to the same as assuming that the value of x "next year" will be equal to the value "this year."

Now let us again suppose that (2) is the a priori theory, but turning to our data, i.e., $x(t)$ ($t=10, 11, \cdots, 61$), we become doubtful. There "must" be a trend, and we change our theory to take account of it. Frequently this is done even if there are no justified a priori arguments for any sort of trend. The "trend" looks fairly linear, and the cycles around it are striking. Suppose, therefore, one assumes the trend to be

$$\bar{x}(t) = kt + b \qquad (k \text{ and } b \text{ constant}). \tag{20}$$

The theory (2) is now assumed to hold in the data adjusted for trend, i.e.,

$$[x(t)-\bar{x}(t)]+a_1[x(t-1)-\bar{x}(t-1)]+a_2[x(t-2)-\bar{x}(t-2)]=0, \tag{21}$$

or, by introducing (20),

$$x(t) + a_1x(t-1) + a_2x(t-2) = c_1t + c_2, \tag{22}$$

where

$$c_1 = (1 + a_1 + a_2)k, \qquad c_2 = (1 + a_1 + a_2)b - (a_1 + 2a_2)k. \tag{23}$$

Now, fitting the equation (22) to the data $x(t)$, ($t=10, 11, \cdots, 61$) by taking the regression of $x(t)$ on $x(t-1)$, $x(t-2)$, and t ($=12, 13, 14, \cdots, 61$), we get

$$x(t) - 0.96x(t-1) + 0.37x(t-2) = 0.031t - 0.71. \tag{24}$$

The fit to the data *within* the observation period is of course here better than, or at least as good as, that obtained by (19), since we now have had two more parameters (c_1 and c_2) at our disposal. Using (23) we obtain:

$$\bar{x}(t) = 0.076t - 1.78 \qquad (t = 12, 13, \cdots, 61). \tag{25}$$

This "trend" is shown in Figure 2.

Solving the homogeneous equation (21) and inserting $a_1=-0.96$, $a_2=0.37$, we get:

$$x(t) - \bar{x}(t) = Ce^{-0.49t} \sin (m + 0.66t), \tag{26}$$

where C and m are arbitrary constants. This is a damped cycle, the period of its harmonic factor being

$$p = \frac{2\pi}{0.66} = \text{about } 9.5 \text{ ``years,''} \tag{27}$$

which is fairly close to the apparent period in the observed series. Although we still should need some external forces to outweigh the damping, since the observed cycle is not systematically damped, we now seem to have some "explanation" of the cycle. In our case we know of course from the construction of the example that this "explanation" has no sense at all. We have obtained this "explanation" by *using the apparent form of the data to formulate our theory. By doing this we have introduced into the theory and explained as structure things which are merely the effect of cumulation of random events.*

Using the equation (24) for forecast one "year" ahead in the same way as we did in the first case, we obtain the curve with cross bars ($t=62, 63, \cdots$) in Figure 2. The formula (24) is clearly useless for forecast purpose. It may be of interest to notice that the coefficients 0.37 and 0.031 in (24) are about three times their standard errors, and the coefficient 0.96 is about seven times its standard error. The coefficient of multiple correlation between $x(t)$ and the other variables is 0.88.

CONCLUSIONS

It seems possible to explain observed cyclical movements by the combination of a structure which is noncyclic, but which contains inertial forces, and outside influences of random events. This possibility should not be excluded a priori; it should be left to be determined by fitting to data.

"Correction" of *the form of* a priori *theory* by pure inspection of the *apparent shape* of time series is a very dangerous proceeding and may lead to spurious "explanations." In particular, the fitting of apparent trends which are not strongly justified on a priori reasons may lead to nonsensical results. Frequently such trend fittings will lead to the conclusion that there are later *changes in structure* (for example during the period $t=65$ to $t=68$ in our constructed example) when the real explanation is the disappearance of spurious elements introduced in our theory by the trend fitting.

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QUANTITY ADJUSTMENT FACTORS IN COST-OF-LIVING RATIOS

By Robert Morse Woodbury

The problem of measuring differences in cost of living has two aspects, that relating to comparisons between differing time periods in a given community or country, and that relating to comparisons between different places or regions at the same time. Cost-of-living formulae have been developed, in general, on the assumption that "tastes, habits and milieu" are the same;[1] this may be substantially valid for comparisons at different times in the same community but does not necessarily hold where different places are compared, and in practice application of the formula to comparison between different places is made subject to the condition that tastes, habits, and milieu are the same or are substantially similar. The question is raised whether a formula for comparing costs of living between places should not explicitly provide a means for taking differences between places, such as those arising from differences in climate, etc., into account. The present paper proposes that a quantity adjustment factor or factors be introduced for this purpose into the cost-of-living formula. In order to clarify terminology the term cost-of-living ratio will be used for formulae or comparisons where the quantity adjustment factor has been taken into account, and the term cost-of-living index where no quantity adjustments have been made.[2]

The usual general formula for cost-of-living comparisons between two places or between two different times—using the usual notation[3]—is

$$I = \frac{\sum p_1 q_1}{\sum p_0 q_0},$$

where the quantities of numerator and denominator relate to equal or identical standards of living. In this form the condition that these standards are equal is essential since the expression merely compares

[1] See, for example, Ragnar Frisch, "Annual Survey of General Economic Theory: The Problem of Index Numbers," Econometrica, Vol. 4, No. 1, January, 1936, pp. 11–12; also, H. Staehle, "A Development of the Economic Theory of Price Index Numbers." *Review of Economic Studies*, Vol. 2, No. 3, June, 1935, p. 164.

[2] Since time comparisons generally do not require quantity adjustments, the current practice of using cost-of-living index for time comparisons corresponds to the terminology proposed; the term cost-of-living ratio can then be used for all comparisons between places, as well as in those few cases of time comparisons, e.g., between summer and winter, where a quantity adjustment factor appears necessary.

[3] I is here used generally as the cost-of-living ratio.

the costs of living (food, fuel, clothing, etc.) in the two situations or times. In general, without reference to whether standards are equal, the ratio of two costs of living in two places or at two times may be set equal to PQ, where P and Q are expressions of relative price and quantity ratios:

$$\frac{\sum p_1q_1}{\sum p_0q_0} = PQ,$$

$$\frac{\sum p_1q_1}{\sum p_0q_0} = \sqrt{\frac{\sum p_1q_0 \sum p_1q_1 \sum p_0q_1 \sum p_1q_1}{\sum p_0q_0 \sum p_0q_1 \sum p_0q_0 \sum p_1q_0}},$$

where the first two expressions under the radical are weighted-average price ratios—the prices, p_0 and p_1 weighted first by the q_0's and then by the q_1's—and the last two expressions are weighted-average quantity ratios—the quantities q_0 and q_1 weighted first by the p_0's and then by the p_1's. If these ratios are designated respectively by P_0 and P_1, Q_0 and Q_1, then

$$\frac{\sum p_1q_1}{\sum p_0q_0} = \sqrt{P_0P_1Q_0Q_1} = PQ$$

where $P = \sqrt{P_0P_1}$ and $Q = \sqrt{Q_0Q_1}$.

$\sqrt{P_0P_1}$ is Fisher's "ideal" formula.

If $Q = \sqrt{Q_0Q_1} = 1$, and if this condition could be taken as meaning that the quantities of the two situations compared relate to equal or identical standards, the cost-of-living ratio between the two situations would be given by the Fisher formula, or $I = \sqrt{P_0P_1}$.

The point of special interest here is that the Fisher formula is an average price-ratio formula, irrespective of the question of equivalence of standards of living. Each element, P_0 and P_1, is a weighted average of price ratios; P_0, for example, can be written

$$\begin{aligned} P_0 &= \frac{\sum p_1q_0}{\sum p_0q_0} \\ &= \frac{1}{\sum p_0q_0}\left[p_0'q_0'\frac{p_1'}{p_0'} + p_0''q_0''\frac{p_1''}{p_0''} + p_0'''q_0'''\frac{p_1'''}{p_0'''} + \cdots\right]. \end{aligned}$$

The usual cost-of-living index formula is thus an average of price ratios and takes no account of any quantity adjustment factor.

The question of quantity adjustment factors in comparisons between different places may be raised in the form of an assumption, the validity of which may then be examined. Assuming that a constant factor is necessary to convert the quantities of one region to equivalent quantities of another to give an identical standard of living, i.e., $\sum q_0$ in re-

gion $0=\sum aq_0$ in region 1, and $\sum q_1$ in region $1=\sum(q_1/a)$ in region 0, then, for the level of living represented by the quantities of region 0,

$$I_0 = \frac{a\sum p_1q_0}{\sum p_0q_0},$$

and for the level of living represented by the quantities of region 1,

$$I_1 = \frac{a\sum p_1q_1}{\sum p_0q_1}.$$

The geometric average of these gives

$$I = \sqrt{I_0I_1} = aP, \quad \text{where} \quad P = \sqrt{P_0P_1}.$$

In effect, if such factors exist, they will appear in the final cost-of-living ratio. The cost-of-living ratio is thus equal to the cost-of-living price index times the conversion factor which is necessary to convert the quantities of one place to those of the other to give equal standards of living. The factor a may be termed the quantity adjustment factor.

This assumption, as thus made, is a highly arbitrary and artificial one, but it may serve to fix attention upon the type of factors which must be taken into account in cost-of-living ratios. So far as their application is concerned, the factors may be applied to cost of living as a whole, or to any of its parts, e.g., cost-of-fuel ratio, cost-of-food ratio, cost-of-clothing ratio.

Fuel presents the clearest evidence for the existence of quantity differences. The amount of fuel required for heating depends obviously upon climate. A dwelling in a warm Southern climate does not require as many heat units of fuel to assure a given degree of comfort as one in a cold Northern region. A fuel quantity budget that is adequate for the former will not be adequate for the latter. A simple average price ratio of the costs of different kinds of fuel, the price of coal in one city compared with the price of coal in another; the price of oil with the price of oil; the price of wood with the price of wood; each price weighted by the respective quantities and a geometric average drawn between the two price ratios, is not a satisfactory cost-of-fuel ratio between the two regions, since it takes no adequate account of these differences in quantities required. A quantity adjustment factor to take account of these differences is evidently an essential part of the cost-of-fuel ratio between two regions of differing climates.

Differences in clothing needs due to climatic differences properly require a factor to take climatic differences into account in a valid cost-of-clothing ratio.

Another illustration may be found in the differences in food require-

ments, for example, between a subtropical and a subarctic region. Calorie requirements in the former are less than in the latter. If the difference in respective calorie requirements can be taken as a measure of the difference in food consumption necessary to maintain equal standards, then the cost-of-food ratio between two such regions will require a factor to express differences in quantity requirements to be used in connection with the usual price ratio.

An interesting case which illustrates the need for a special factor is the difference in food requirements for peoples of different average stature. Calorie requirements for short-statured peoples like the Filipinos and the Japanese are less than those for Europeans. Entirely apart from differences in taste, etc., differences in average stature or size will produce differences in food requirements to maintain the same "standard" of food consumption.[4]

If these examples may suffice to establish the existence of quantity-differential factors, they suggest that a formula in which the quantity differential is taken as a constant is too simple. The adjustment required for fuel is scarcely the same as that for clothing, or for food, or for miscellaneous items. Even the adjustment for different items of fuel and light may be different.

A more general formula, therefore, to take account of variations in the quantity adjustment factor in different parts of the budget or for individual items may be developed. Let r_0q_0 be the quantity in the 1-situation equivalent to q_0 in the 0-situation and q_1/r_1 the quantity in the 0-situation equivalent to q_1 in the 1-situation.

In this case, starting with the quantities in the 0-situation,

$$I_0 = \frac{\sum p_1q_0r_0}{\sum p_0q_0},$$

where r_0 takes successively the values of r_0', r_0'', r_0''', etc., corresponding to q_0', q_0'', q_0''', etc.

This may be written

$$I_0 = \left(\frac{\sum p_1q_0r_0}{\sum p_0q_0r_0}\right)\left(\frac{\sum p_0q_0r_0}{\sum p_0q_0}\right).$$

[4] In practice, however, this type of special factor is rarely needed, since few studies attempt to set up a cost-of-food ratio to compare the cost of food of a Japanese living in Japan and of a European living in Europe. The index is usually the cost-of-food ratio between a Japanese living in Japan and a Japanese living in Europe, or between a European living in Japan and one living in Europe. But if the cost-of-food ratio sought between Japan and Europe is between a Japanese living in Japan and a European living in Europe, a special factor must obviously be introduced to take account of the differences in calorie requirements due to differences in average size.

The second ratio is a weighted-average quantity adjustment factor,

$$R_0 = \frac{\sum p_0q_0r_0}{\sum p_0q_0},$$

weighted by the quantities times the prices of region 0. The first ratio may be designated by P_0^*. It is a weighted-average price ratio, using as weights the quantities of region 0 as modified by the adjustment factors.

The corresponding value, starting from the quantities in the 1-situation, is

$$I_1 = \frac{\sum p_1q_1}{\sum p_0\left(\frac{q_1}{r_1}\right)} = \frac{\sum p_1\left(\frac{q_1}{r_1}\right)}{\sum p_0\left(\frac{q_1}{r_1}\right)} \frac{\sum p_1q_1}{\sum p_1\left(\frac{q_1}{r_1}\right)},$$

which may be written $I_1 = P_1^*R_1$ where

$$R_1 = \frac{\sum p_1q_1}{\sum p_1\left(\frac{q_1}{r_1}\right)}$$

and P_1^* is the weighted-average price ratio using the adjusted quantities of region 1.

Taking the geometric average of the two indices, $I = \sqrt{P_0^*P_1^*R_0R_1}$, which may be written

$$I = P^*R,$$

where $P^* = \sqrt{P_0^*P_1^*}$ and $R = \sqrt{R_0R_1}$.

If $R_0 = R_1$, or if the difference between them is small, P^* may be taken as equal to $P = \sqrt{P_0P_1}$, since the quantities used in P_0 and P_1 are modified in approximately equal proportions and in opposite directions to form the quantities used in P_0^* and P_1^*.

Instead of the general formula,

$$\frac{\sum p_1q_1}{\sum p_0q_0} = PQ$$

we may now write

$$\frac{\sum p_1q_1}{\sum p_0q_0} = P^*RS$$

where R represents (one or more) quantity adjustment factors, and S may be regarded as the ratio of the costs—after elimination of price

differences and correction for quantity adjustments—of the two standards of living corresponding to the numerator and denominator respectively.[5]

The formula for the cost-of-living ratio, then, consists of two parts, one to measure the average price ratio and the other a quantity adjustment factor to measure the average ratio of the quantities required in the two situations to yield equal standards. It remains to be considered by what means these quantity adjustment factors can be calculated.

The mathematical formulations of cost-of-living index numbers appear, at first glance, to offer means of determining, if not quantity correction factors, then cost-of-living ratios including both price and quantity correction elements.[6] The mathematical or functional approach starts with the concept of identical standards of living rather than with identical quantities in the two situations. If in the formula just developed, the r's be regarded not as a factor to take account specifically of climate, etc., but as a general factor to cover all elements required to produce equivalent standards in the two situations, it conforms closely with the general mathematical formulation of the problem of cost-of-living comparisons. The difficulty is to develop the mathematical concepts in such a way that they lead to practicable methods for calculating the ratios.

Frisch's so-called "double-expenditure" method is one of these math-

[5] The following shows the exact meaning of each of these factors.

$$\frac{\sum p_1q_1}{\sum p_0q_0} = \sqrt{\frac{\sum p_1q_0 \sum p_1q_1 \sum p_0q_1 \sum p_1q_1}{\sum p_0q_0 \sum p_0q_1 \sum p_0q_0 \sum p_1q_0}} = \sqrt{P_0P_1Q_0Q_1} = PQ$$

$$= \sqrt{\frac{\sum p_1q_0r_0 \sum p_1\left(\frac{q_1}{r_1}\right) \sum p_0q_0r_0 \sum p_1q_1 \sum p_0\left(\frac{q_1}{r_1}\right) \sum p_1q_1}{\sum p_0q_0r_0 \sum p_0\left(\frac{q_1}{r_1}\right) \sum p_0q_0 \sum p_1\left(\frac{q_1}{r_1}\right) \sum p_0q_0 \sum p_1q_0r_0}}$$

$$= \sqrt{P_0{}^*P_1{}^*R_0R_1S_0S_1}$$

$$= P^*RS.$$

S is the geometric average of two values, S_0 and S_1, each representing the ratio between the values found by multiplying by the prices of the region the quantities of the region itself and the quantities of the other region as modified by the adjustment factor.

The adjustment factor is an essential part of S, but though it appears also in P^*, its role there is of minor importance and P^* in practice agrees closely with P. If this is true, we may say that for practical purposes $Q = RS$.

[6] In the mathematical discussions, however, no special account is taken of the type of quantity adjustment factors here under discussion, in fact in some cases such factors appear to be explicitly excluded from consideration.

ematically derived formulations.[7] Where the quantities in the two situations satisfy the criterion that the double expenditures are equal, the ratio of the costs of living gives, it is claimed, a true cost-of-living ratio. But this criterion is none other than $Q=\sqrt{Q_0Q_1}=1$,[8] and as developed above, if this holds, the true index is identical with the price-ratio index given by the Fisher ideal formula.[9]

In other words, the condition that double expenditures are equal fails to take account of any quantity adjustment factors. The true condition that standards of living are equal would appear to be $S=1$ rather than $Q=1$ and the desired cost of living ratio is then P^*R. Unfortunately, while the condition that $Q=1$ can be utilized to calculate a definite ratio, before the condition that $S=1$ can be utilized, some method must be found to calculate R.

It may perhaps be suggested that an alternative conclusion might be that no quantity adjustment factor is necessary. Such a conclusion cannot be accepted. So far as the cost-of-fuel ratio between two places

[7] Ragnar Frisch, "Annual Survey of General Economic Theory: The Problem of Index Numbers," ECONOMETRICA, Vol. 4, No. 1, January, 1936, p. 29; also, "The Double-Expenditure Method," ECONOMETRICA, Vol. 6., No. 1, January, 1938, p. 85. The double-expenditure criterion is

$$(\sum p_1q_1)(\sum p_0q_1) = (\sum p_1q_0)(\sum p_0q_0).$$

[8] $$\sqrt{Q_0Q_1} = \sqrt{\frac{\sum p_1q_1 \sum p_0q_1}{\sum p_1q_0 \sum p_0q_0}} = 1.$$

Squaring and rearranging:

$$\sum p_1q_1 \sum p_0q_1 = \sum p_1q_0 \sum p_0q_0.$$

[9] The same conclusion may be reached in another way. Using the notation of Frisch's article, at the point q_1^*, where the quantities in the 1-situation are equivalent to q_0 of the 0-situation, the "true" ratio is

$$I = \frac{\sum p_1q_1^*}{\sum p_0q_0}.$$

At the point of equivalence, the double-expenditure criterion (approximately) holds, namely:

$$\sum p_0q_0 \sum p_1q_0 = \sum p_0q_1^* \sum p_1q_1^*$$

$$\frac{\sum p_1q_0}{\sum p_0q_1^*} = \frac{\sum p_1q_1^*}{\sum p_0q_0}.$$

Multiplying both sides by the right-hand side and extracting the square root:

$$\sqrt{\frac{\sum p_1q_0 \sum p_1q_1^*}{\sum p_0q_0 \sum p_0q_1^*}} = \frac{\sum p_1q_1^*}{\sum p_0q_0}.$$

The left-hand side is the Fisher ideal formula, while the right-hand side is the "true" index, at the point of equivalence. (I am indebted to Dr. Walter Kull for this note.)

with differing climates is concerned, the weighted-average price ratio for a budget of fuel does *not* give a sound cost-of-fuel ratio where climatic differences are important. If this is true of part of the budget, the only way in which the whole budget can dispense with a quantity adjustment factor is in case the different quantity adjustment factors cancel out, i.e., if more must be spent for fuel, less must be spent for something else, the same standard of living being maintained. But if climate requires that larger expenditures for fuel be made, it is probable that more must be spent for clothing also, and perhaps more for food; correspondingly less would be available for miscellaneous items, and the standard of living would be impaired, if the quantity adjustment factors had to cancel out in the budget as a whole.

The truth appears to be that the double-expenditure criterion is valid only when $R=1$ and that it is a price-ratio formula. Hence, where quantity adjustment factors are important, it will not give a satisfactory cost-of-living ratio.[10]

In default of a satisfactory mathematical solution other ways of determining these factors may be considered. Some hints of possible methods have already been given. In general, two modes of attack are available, the first, to determine the value of the quantity adjustment factor itself, to be used with the price ratio to form a cost-of-living ratio, and the second to determine directly the costs of the items to be compared, from which the cost-of-living ratio in respect of the particular items concerned can be calculated.

So far as the first method is concerned, we have in the case of food the possibility of using physiological determinations of calorie requirements under different conditions as a basis for quantity adjustment factors. For fuel, quantity adjustment factors might be calculated in terms of differences in heat units required for room heating in climates of varying outdoor temperatures, leading to a formula expressing the adjustment factor in terms of average monthly temperatures for the

[10] Another mathematical formulation is that of Bowley, who recommends the weighted-average price ratio:

$$\frac{\sum p_1(q_0 + q_1)}{\sum p_0(q_0 + q_1)} .$$

A. L. Bowley, "Notes on Index Numbers," *Economic Journal*, Vol. 38, 1928, pp. 216–237.

Still another formulation is that of Wald, which reduces to a similar formula, in which the q_1's are, however, multiplied by a special factor. A. Wald, "New Formula for the Index of Cost of Living," ECONOMETRICA, Vol. 7, No. 4, October, 1939, pp. 319–331. The special factor is the ratio of the marginal utilities of money corresponding to the two points, $\bar{q}_1$ and q_1 in the 1-expansion path.

Obviously neither of these contains any quantity-adjustment factors. Wald's formula is developed explicitly for cost-of-living indices at different times.

winter months. Such a solution would lend itself admirably to adjustment for local climatic conditions. In the case of gas used for cooking, etc., the number of months of such use could be utilized as a basis for a quantity adjustment factor. A similar method taking into account the number of months of warm weather might be applied to ice.[11] Quantity adjustment factors for clothing, bedding, etc. offer special problems for determination but are relatively of little importance.[12]

The second method is to determine by direct investigation the cost of the adjusted items in the two situations. This offers certain difficulties and opens the way to arbitrary determinations. Difficulties arise in ensuring that the costs compared relate really to the same standard of living. This method was followed, for example, in the International Labour Office study comparing costs of living at a given level—that for Ford employees—in 14 European cities as compared with Detroit. In this study, quantity adjustments were restricted to fuel, and the cost of fuel in European cities at the given level of living was simply compared with the cost in Detroit. The quantities of fuel required naturally varied from city to city, the actual amounts being usually determined on the basis of family-budget studies or local-family consumption.[13]

In two cases special procedures were followed to aid in obtaining the

[11] See United States Works Progress Administration, Research Monograph XII, *Inter-city Differences in Costs of Living, in March 1935, 59 cities*, Margaret Loomis Stecker, pp. 42–52, also 106–111. In this study, fuel adjustment factors were derived for 4 groups of cities grouped according to an appraisal of their climate as (A) winters long and/or cold; (B) average; (C) short and/or mild; (D) very short and/or very mild. The factors were, setting the B group = 1.00: A, 1.154; B, 1.000; C, 0.615; D, 0.269. The number of months during which coal or wood was used as fuel was one element in determining these factors but not the sole element. The exact method followed is not clearly indicated; presumably it was based upon estimates of fuel required, derived from a special enquiry (*ibid.*, p. 147). The adjustment factor can be applied to the heat units required in terms, for example, of British thermal units—the unit is the amount of heat required to raise 1 lb. of water 1 degree Fahrenheit. By this means the heat value of different kinds of gas, natural gas, manufactured gas, can be expressed in terms of a common unit. The same method may also be applied to coal and wood, using heat value as a basis of comparison.

[12] An adjustment factor for rent might be calculated by finding the average ratio of the costs in each community of the types of dwelling corresponding respectively to the requirements in the two regions, e.g., taking account of differences in housing construction, insulation, and heating equipment, etc., required to meet climatic differences; in practice, because of the great difficulties in obtaining the rent ratio, the rents of comparable dwellings in the two regions allowing for the differences in construction associated with climate are compared directly (the second method).

[13] International Labour Office, Studies and Reports, Series N, Statistics, 17, *A Contribution to the Study of International Comparisons of Costs of Living.*

desired equivalent standards. In the case of Ireland, the percentages of total expenditure spent for fuel were found to be nearly constant: the general expenditure required in Ireland corresponding to the desired level in Detroit having been determined, the corresponding fuel budget was determined by means of this percentage.[14] In the case of Germany, the fuel cost per room was found to be nearly constant. The average size of dwelling in Detroit having been found, the corresponding cost of fuel in Germany for a dwelling of the same size was then calculated.[15]

An adjusted rent ratio comparing in two communities rents of houses alike in all respects save for the differences in construction, heating equipment, etc., associated with climatic differences can be calculated directly. This was done, for example, in a study comparing rents of houses in five Northern communities in the United States equipped with furnaces with those of houses in five Southern communities equipped with gas heaters, the method of construction in each region being that customarily prevailing; other elements, however, number of rooms, other equipment, etc., being the same.[16] This method of comparing adjusted costs might prove useful also for calculating adjusted cost-of-clothing ratios, though in this case difficulties arise in ensuring a common standard for comparison.

The particular method to be adopted depends in part upon the exact meaning to be assigned to the quantity adjustment factor. A method of utilizing figures of actual average consumption in the two regions suffers from the objection that such figures are affected by differential efficiency, waste, differences in custom, as well as differences in average level of living. They can be accepted, therefore, as a basis for measuring quantity adjustment factors only on the assumption that relative efficiency of consumption remains about the same and that differences in the average degree of comfort are not important. The same objection is applicable to methods based upon actual average costs—average quantities consumed multiplied by local prices. These also may contain elements of waste, differential efficiency, customs, or differences in standards.[17] On the other hand, a method based upon a formula relat-

[14] *Ibid.*, p. 20. A modification of this would be to utilize the ratio between cost of fuel and cost of rent as a method of estimate of fuel costs in relation to the standard of living represented by the rent item (proposal of Mr. Robert Guye).

[15] *Ibid.*, p. 20.

[16] See "Differences in Living Costs in Northern and Southern Cities," *Monthly Labor Review*, Vol. 49, July, 1939, p. 33.

[17] A special quantity factor might be introduced to take account of waste or inefficiency in the use, for example, of fuel. If in one region, owing to inadequate heating equipment, twice the quantity of fuel were used as was necessary with proper equipment to yield the degree of comfort desired, a waste factor would

ing the adjustment factor to average winter temperatures could easily eliminate any such differences.[18]

How important are these quantity adjustment factors? In other words, if a price-ratio index is used instead of a cost-of-living ratio, what percentage error may be ascribed to the absence of a quantity correction factor? The answer to this question depends obviously upon the differences between the two communities compared and the importance of the items to be corrected. A study of comparisons of costs of living in five Northern and five Southern cities in the U. S. throws light upon this point.[19] The average net error in the ratio which would have been caused by the neglect of quantity adjustments in regard to fuel alone may be estimated at 3.0 per cent. Though in any given comparison the value of the correction must depend upon the particular conditions, it may nevertheless be concluded that neglect of these factors may cause an appreciable error in cost-of-living comparisons.

CONCLUSIONS

(1) The cost-of-living ratio between two regions as distinguished from the cost-of-living index between two times requires or may require quantity adjustment factors to take account of the differences in requirements for maintaining equivalent standards in the two regions.

(2) A formula for the quantity adjustment factor is developed.

(3) Mathematical analyses of the cost-of-living index number problem do not as yet appear to provide a satisfactory method for evaluating quantity adjustment factors.

(4) A number of possible methods are suggested, some of which have been used in measuring, for example, the fuel adjustment ratio, in the few studies devoted to this problem.

(5) The aim of the present paper has been to clarify the theoretical position and justification for quantity adjustment factors and to call attention to the need for better methods for their determination.

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call attention to differences in expense that corresponded neither with differential requirements nor with differences in living standards. (Conspicuous waste might be allocated to a difference in standard.)

[18] If preferred, average differences in efficiency, in the case of fuel for example, corresponding to prevailing types of heating equipment in use in the different regions, might be ignored.

[19] "Differences in Living Costs in Northern and Southern Cities," *Monthly Labor Review*, Vol. 49, No. 1, July, 1939, pp. 22–38, especially 25–26.

RECURSIVE METHODS IN BUSINESS-CYCLE ANALYSIS[1]

By Merrill M. Flood

1. THE PROBLEM OF TAX REVENUE ESTIMATION

The "business cycle" is a different being for each person discussing it. This paper deals not with generalities of "business-cycle" theory but with a very particular problem, that of forecasting tax revenue yields for the State of West Virginia. This is surely a problem encompassed by that branch of economics known as business-cycle theory.

The receipts of the West Virginia General Revenue Fund are derived from:[2]

1. A comprehensive gross-income tax on production, sales, and service.
2. A broad consumer's sales tax on all business.
3. Business profits of the West Virginia Liquor Control Commission.
4. A gross-receipts tax on public utilities.
5. A personal net-income tax.
6. A tax on insurance premiums.
7. A barrel tax on beer sales.
8. An inheritance tax.
9. Miscellaneous fees and license taxes.

Revenue yields from these sources vary directly with the course of business so the problem of revenue estimation and budget control in West Virginia is essentially the problem of the business cycle.

To be of any value, revenue forecasts must be in dollars and they must be accompanied by standard errors of estimate. For this purpose it is essential that mathematical forecasting formulas be developed for each tax and that these formulas be tested over a period not used in their construction, in order that suitable standard errors of estimate may be available. This paper presents the methods used to accomplish these results and discusses the accuracy and reliability of the business-cycle forecasting formulas obtained.

In general, the empirical method was used throughout. The basic assumption has been that each series studied satisfies a linear stochastical difference equation. Considerable theoretical justification for this approach is afforded by the studies of Tinbergen,[3] and many

[1] Read before a joint session of the Econometric Society and the Institute of Mathematical Statistics at Philadelphia, December 28, 1939.

[2] For a history of the yields of these taxes see the West Virginia *Budget Document*, 1939, p. 43.

[3] For a discussion of this type of approach to business-cycle theory, and reference to other authors see J. Tinbergen, Econometrica, Vol. 3, July, 1935, pp. 241–308, and Vol. 6, January, 1938, pp. 22–39.

others. No further attempt was made to provide theoretical justification for the use of this type of mathematical model to represent the business-cycle phenomenon, but every effort was made to test the results obtained against actual experience.

The standard errors of forecast may seem rather large. Large is a comparative term, however, and until other forecasting formulas are developed which are superior to the ones presented here we must be content with them. For our immediate problem of budget control the formulas obtained have proved to be quite satisfactory.

Perhaps the greatest obstacle encountered in this type of empirical analysis is the lack of a norm with which the results obtained by these methods may be compared. It may well be that the accuracy of our forecasts would be greatly improved if we were to use another approach but, so far as I know, there is no suitable means of comparing the vast number of business-cycle theories without trying them all on some particular series to determine which works the best. This would provide only a single clue but would be more than we now have. It would be a fine thing if all inventors of methods for constructing a business-cycle theory to explain a particular time series would try their methods out on the same series, say monthly steel-ingot production in the United States. There are probably better ways for obtaining such a practical comparison of methods but my plea is that students of business-cycle theory adopt one so that we may have at least a crude quantitative measure of the degree of our progress from time to time.

2. A SIMPLIFICATION

After some study we found that the several West Virginia revenue series which we desired to forecast followed very much the same pattern over the period for which data were available. After account had been taken of the different lags, relationships were obtained which expressed each of the series in terms of a single business series.[4] This accomplished, there remained only the problem of forecasting the indicator series chosen.

In order that data might be available for analysis for a period longer than that available for any of these taxes a nontax series was chosen as the indicator series. This series is the monthly amount of Total Orders Entered by the Westinghouse Electric and Manufacturing Company. This series was used, not only because it was found possible to express the several tax series rather accurately in terms of it, but also because a business-cycle theory had been developed for this series by F. D. Newbury,[5] Economist for Westinghouse, who was kind enough

[4] West Virginia Revenue Series, No. 3, "Forecasting the General Revenue Fund," Charleston, September, 1939 (mimeo.).

[5] F. D. Newbury, *Westinghouse Forecasting Method* (mimeo.).

to provide us with the data and a statement of his approach to the forecasting problem. The forecasts made by the Westinghouse executives also provide a norm against which to test our own results. This Westinghouse Orders Series was then investigated in some detail.

3. THE MATHEMATICAL MODEL USED

Before presenting the results of this rather exhaustive, and exhausting, empirical analysis a description of the mathematical methods used is in order. The basic mathematical questions involved are treated in some detail by H. Wold[6] in his recent book, to which reference is made for a more complete discussion of this problem.

We may denote the time series under consideration by $w(t)$ for $t=1, 2, \cdots, n$. From this series others may be derived in many ways —for example, by averaging processes, differencing, percentages of corresponding previous time intervals, and so forth. Our empirical approach demands that such a derived series $W(T)$ be constructed which is believed to satisfy a linear stochastical difference equation, of prescribed order, with coefficients essentially independent of time T. By application of the usual methods of regression theory these coefficients are then estimated, after the assumption of some probability distribution for the deviations, and the resulting formula is tested by application to subsequent data not used in the regression analysis.

The linear difference equation provides a more general mathematical mechanism for this analysis than may be realized by the nonmathematician. If the difference equation is

$$W(T) + a_1W(T-1) + \cdots + a_vW(T-v) = 0,$$

its "characteristic equation" is defined as

$$x^v + a_1x^{v-1} + \cdots + a_{v-1}x + a_v \equiv \prod_{k=1}^{r}(x-p_k)^{m_k}\prod_{k=1}^{j}(x^2+2s_kx+q_k^2)^{n_k} = 0,$$

where p_k, s_k, and q_k are real. Then the general solution of the difference equation is[7]

$$W(T) = m + \sum_{k=1}^{r} H_{m_k}^{1k}(T)p_k^T + \sum_{k=1}^{j}\left[H_{n_k-1}^{2k}(T)\cos\lambda_kT + H_{n_k-1}^{3k}(T)\sin\lambda_kT\right]q_k^T,$$

where m is an arbitrary constant, H_h^{ik} is an arbitrary polynomial of order h, and $\cos\lambda_k = -s_k/q_k$. The solutions are therefore seen to be very

[6] H. Wold, *A Study in the Analysis of Stationary Time Series*, Uppsala, 1938.
[7] H. Wold, *op. cit.*, pp. 19–21.

general functions indeed. For example, the difference equation for $W(T) = \sin T$ is simply

$$\sin T - (2 \cos 1) \sin (T - 1) + \sin (T - 2) = 0,$$

which is obtained from the general solution above by setting $j=1$, $m=0$, $r = 0$, $n_1 = 1$, $H_0^{21} = 0$, $\lambda_1=1$, $q_1 = 1$, and $H_0^{31} = 1$.

When, by empirical analysis, such a linear difference equation is determined to represent the time series under consideration, its solution is used for forecasting purposes by choosing from among the arbitrary coefficients the ones which yield a function which passes through the initial points chosen. As the set of initial points moves along so does the function change as to phase and amplitude, but not as to length of cycles. It is this property of shifting phase and changing amplitude which makes the difference equation, used in this fashion, a very general tool for analysis and prediction. The classical case of strict harmonic analysis is included as the special case where the initial points are taken to be independent of T. Then the data are fitted functionally by the difference equation and the set of initial points chosen. These various cases are illustrated in the discussion which follows.

4. ESTIMATION OF THE PARAMETERS

From an observational time series a difference equation of prescribed order may be derived by using the method of least squares. If v denotes the order, and r the number of time intervals forecast, this equation may be written

$$W(T + r - 1) = a_{r1}W(T - 1) + a_{r2}W(T - 2) + \cdots + a_{rv}W(T - v).$$

If the coefficients are obtained for $(r=1, 2, 3, \cdots, p)$, then this set of formulas will forecast from 1 to p time intervals ahead.[8]

Another method of obtaining forecasting difference equations depends on the use of the "serial coefficients," as defined by G. U. Yule.[9] If $W(T)$, for $(T=1, 2, \cdots, N)$, denotes an observational time series, its serial coefficients R_K are the correlation coefficients between the two series $W(T)$ and $W(T+K)$ for $(K=0, 1, 2, \cdots, N-1;\ T=1, 2, \cdots, N-K)$. In similar manner, the "autocorrelation coefficients" of a function $W(T)$ are the correlation coefficients between $W(T)$ and $W(T+K)$, as above. Since there is a unique linear difference equation of order v whose functional solutions have prescribed autocorrelation coefficients $R_1, R_2, \cdots, R_v$, we may determine such an equation

[8] Formulas 1–5 and 23–32 of Table 1 are of this type.

[9] G. U. Yule, *Journal of the Royal Statistical Society*, Vol. 89, 1926, pp. 1–64.

by assuming that these first v autocorrelation coefficients are the first v serial coefficients obtained from the observational time series above.[10]

Yet another method of obtaining estimates of the parametric coefficients which enter into the difference equations being studied is to assume that the deviations have a nonnormal distribution $ke^{-c|x|}$ and obtain the coefficients by minimizing the absolute value sum rather than the sum of squares. This was carried out for one series, using a method developed by R. R. Singleton,[11] and the resulting forecasts

TABLE 1

SUMMARY OF FORECASTING FORMULAS

Formula No.	Method of Derivation	Data Period				Testing Period			
		Time Interval	Mean ($000,000)	Std. Dev. ($000,000)	Coeff. of Var. %	Time Interval	Mean ($000,000)	Std. Dev. ($000,000)	Coeff. of Var. %
1–5	Least Squares	3/23–12/36	1523	459	30.0	3/37–9/39	1930	520	26.9
6	Serial Coeff.	3/23–12/36	1523	459	30.0	3/37–9/39	1930	520	26.9
9–16	Recursion #5	3/23–12/36	1523	459	30.0	3/37–9/39	1930	520	26.9
7– 8	Recursion #6	3/23–12/36	1523	459	30.0	3/37–9/39	1930	520	26.9
17	Absolute Value	12/33–12/36	1207	302	25.0	3/37–9/39	1930	520	26.9
23–27	Least Squares	6/31–12/36	746	131	17.6	3/37–9/39	1027	291	28.3
18–22	Recursion #3	3/23–12/36	1523	459	30.0	3/37–9/39	1930	520	26.9
28–32	Least Squares	9/31– 3/37	750	141	18.8	3/37–9/39	1027	291	28.3
33–37	Recursion #23–27	6/31–12/36	746	131	17.6	3/37–9/39	1027	291	28.3

(1) $W(t)=1.368\ W(t-3)-0.488\ W(t-6)-0.357\ W(t-9)+1.362\ W(t-12)-0.945\ W(t-15)$.
L.S. 9/25–6/35. Data % = 5, Testing % = 39. Real root = 0.98.
Cycles: 41.6 and 9.2 months.

(2) Formula 5, Cycles: 8.6 and 46.5 months.

(3) Whittaker Periodogram: 9 and 40 months from annual Westinghouse.
Data: 3/23–12/36. 9 and 38 months from monthly Westinghouse.

(4) $W(t)=1.641W(t-1)\quad -0.221W(t-2)\quad -0.297W(t-3)\quad -0.727W(t-4)\quad +0.585W(t-5)$ $+0.408W(t-6)\quad -0.367W(t-7)$.
L.S. 3/32–12/36. Data % = 5, Testing % = 7.2. Real roots: 1.08, 0.85, −0.66.
Cycles: 8.7 and 28.9 months.

were more accurate than those obtained by the other methods discussed.[12] This test was not conclusive, since it includes but one such test, but the most interesting characteristic of the result was that the difference equation so obtained was "explosive" in character, rather than "damped." The practical difficulties of computation again precluded our making an adequate test of this method.

For forecasts of more than one time interval formulas may be obtained by recursion from those predicting one time interval, whether obtained by the regression methods or the method of serial coefficients described above; and whether obtained on the hypothesis of a normal

[10] This method has been suggested by Wold. Formula 6 of Table 1 was obtained in this manner.

[11] R. R. Singleton, unpublished.

[12] This example is presented in the Tables 3 and 4.

TABLE 2

FORECASTING EQUATIONS AND STANDARD DEVIATIONS

Formula No.	r	Coeff. of Var. Data (%)	Coeff. of Var. Test (%)	Serial Std. Dev. Data ($000,000)	Serial Std. Dev. Test ($000,000)	Regression Std. Dev. Data ($000,000)	Regression Std. Dev. Test ($000,000)	Coefficients of $t-1$	Coefficients of $t-2$	Coefficients of $t-3$	Coefficients of $t-4$	Coefficients of $t-5$
						Westinghouse Total Orders Entered						
1	1	7.6	....	115	203	115	...	1.003	.....	.....	.....	.....
2	1	5.3	7.0	115	203	80	135	1.742	−0.744	.....	.....	.....
3	1	5.3	7.2	115	203	81	138	1.713	−0.674	−0.041	.....	.....
4	1	5.4	....	115	203	81	...	1.713	−0.665	−0.064	0.014	.....
5	1	4.7	7.1	115	203	71	138	1.704	−0.636	0.266	−0.841	0.507
6	1	....	8.0	115	203	...	156	1.633	−0.581	−0.090	−0.108	0.095
7	2	....	17.0	213	386	...	322	2.087	−1.038	−0.255	−0.081	0.155
8	3	....	27.0	302	534	...	522	2.371	−1.467	−0.269	−0.070	0.198
9	2	10.0	17.0	213	386	142	326	2.267	−0.818	−0.387	−0.926	0.865
10	3	15.0	26.0	302	534	225	511	3.045	−1.830	−0.322	−1.042	1.150
11	4	22.0	33.0	381	640	328	641	3.358	−2.260	−0.231	−1.411	1.545
12	5	27.0	38.0	445	710	402	736	3.463	−2.368	−0.516	−1.280	1.704
13	6	31.0	42.0	499	745	466	825	3.532	−2.720	−0.358	−1.209	1.757
14	7	35.0	45.0	543	762	519	880	3.299	−2.605	−0.268	−1.214	1.793
15	8	38.0	46.0	583	757	567	888	3.015	−2.367	−0.335	−0.982	1.674
16	9	40.0	....	619	758	611	...	2.771	−2.254	−0.179	−0.863	1.530
17	1	....	13.0	...	...	...	271	0.873	0.144	0.469	−0.936	0.547
18	2	11.0	22.0	213	386	161*	334	2.259	−1.195	−0.069	.....	.....
19	4	21.0	46.0	381	640	326*	701	2.985	−1.894	−0.108	.....	.....
20	6	32.0	61.0	499	745	482*	927	3.391	−2.292	−0.130	.....	.....
21	8	41.0	57.0	583	757	624*	887	3.605	−2.509	−0.143	.....	.....
22	10	49.0	52.0	655	779	750*	806	3.704	−2.618	−0.149	.....	.....
						West Virginia Article 13 Gross Sales Tax						
23	1	6.1	4.5	44	46	45	46	1.000	.....	.....	.....	.....
24	1	2.9	3.2	44	46	22	32	1.886	−0.882	.....	.....	.....
25	1	3.0	3.1	44	46	23	32	1.825	−0.751	−0.070	.....	.....
26	1	3.1	3.0	44	46	23	31	1.832	−0.623	−0.370	0.165	.....
27	1	3.1	3.1	44	46	23	32	1.814	−0.584	−0.297	−0.047	0.118
28	2	11.7	....	86	87	88	...	1.003	.....	.....	.....	.....
29	2	6.1	7.6	86	87	46	78	2.681	−1.671	.....	.....	.....
30	2	6.3	7.7	86	87	47	79	2.724	−1.765	0.050	.....	.....
31	2	6.3	7.3	86	87	47	75	2.740	−1.448	−0.692	0.408	.....
32	2	6.5	7.5	86	87	48	77	2.716	−1.397	−0.594	0.125	0.157
33	2	8.5	....	86	87	64*	...	1.000	.....	.....	.....	.....
34	2	6.3	....	86	87	47*	...	2.674	−1.664	.....	.....	.....
35	2	6.3	....	86	87	47*	...	2.580	−2.071	−0.128	.....	.....
36	2	6.4	....	86	87	48*	...	2.732	−1.512	−0.513	0.302	.....
37	2	6.4	....	86	87	48*	...	2.705	−1.357	−0.585	0.032	0.213

r = Number of quarters prediction.

* Derived theoretically from original one-quarter prediction formula and variance.

TABLE 3

LINEAR REGRESSION COEFFICIENTS
FIT BY ABSOLUTE VALUES FOR ESTIMATION OF $W(t)$
BY FIVE PRECEDING OBSERVATIONS

Number of Observations	Coefficients of				
	$W(t-1)$	$W(t-2)$	$W(t-3)$	$W(t-4)$	$W(t-5)$
10	1.048	0.343	−0.761	−0.332	0.819
11	1.048	0.343	−0.761	−0.332	0.819
12	1.543	−0.298	0.298	−1.203	0.684
13	0.873	0.144	0.469	−0.936	0.547
Least-Squares Fit, for Comparison					
10	1.151	0.766	−0.217	−0.575	0.664

TABLE 4

COMPARISON OF LINEAR DIFFERENCE EQUATIONS OBTAINED BY MINIMIZING SUM OF SQUARES AND SUM OF ABSOLUTE VALUES
(5 Coefficients, 10 Observations)

Characteristic Equation with Real Root (Trend) Removed:	L.S.	A.V.
x^4	1	1
x^3	−0.081	0.014
x^2	−0.163	−0.328
x	0.042	0.413
1	0.620	0.771
Quarterly Exponential Trend (Real Root)	1.071	1.062
Major Period (Quarters)	8.5	8.0
Modulus of Root	0.911	1.027
Minor Period (Quarters)	2.6	2.4
Modulus of Root	0.865	0.867
Standard Deviations:		
Data Period	61	68
Test Period	1521	253

distribution, the nonnormal distribution of the previous paragraph, or some other. Thus, if $W(T)$ is assumed to satisfy the linear stochastical difference equation

$$W(T) = a_1W(T-1) + a_2W(T-2) + \cdots + a_vW(T-v),$$

then it should also satisfy the equation

$$W(T+1) = (a_1^2 + a_2)W(T-1) + (a_1a_2 + a_3)W(T-2) + \cdots + (a_1a_{v-1} + a_v)W(T-v+1) + a_1a_vW(T-v).$$

More generally, we may set

$$\begin{Vmatrix} W(T) \\ W(T-1) \\ W(T-2) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ W(T-v) \end{Vmatrix} = \begin{Vmatrix} a_1 & a_2 & a_3 \cdots a_{v-2} & a_{v-1} & a_v \\ 1 & 0 & 0 \cdots 0 & 0 & 0 \\ 0 & 1 & 0 \cdots 0 & 0 & 0 \\ \cdot & \cdot & \cdot\ \cdot\ \cdot\ \cdot & \cdot & \cdot \\ 0 & 0 & 0 \cdots 0 & 1 & 0 \end{Vmatrix} \begin{Vmatrix} W(T-1) \\ W(T-2) \\ W(T-3) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ W(T-v-1) \end{Vmatrix}$$

or, in abbreviated form,

$$W_T = MW_{T-1}.$$

Then, by successive applications of this relation, we obtain

$$W_{T+k} = M^k W_T,$$

which is a formula forecasting k time intervals ahead but obtained from one forecasting only one time interval ahead.[13]

5. TESTING THE FORECASTING FORMULAS

In testing these various forecasting formulas use was made of the variance between the observed and predicted values. A testing period, different from the data period used in the derivation of the formulas, was used in determining the testing variances. In the case of the forecasting formulas obtained by recursion there are three variances to be considered, one from the data period, one derived theoretically[14] from the parameters of the original difference equation, and one from the testing period. In each case we placed the greatest confidence in the testing-period variance, particularly when it was significantly greater than the theoretical-data-period variance.

To provide a means of comparison of forecasting accuracy of several formulas with different data and test periods, the coefficient of variability was used, that is, the ratio of the standard deviation to the mean of the series in the period under consideration. These coefficients serve, also, as rough indications of the absolute accuracy of the forecasting formulas.[15]

Probably the best test of the efficiency of a particular forecasting formula is the extent of the reduction of variance. If, by the "regression deviation" σ is meant the standard deviation[16] of the actual from the

[13] Formulas 7–16, 18–22, and 33–37 of Table 1 illustrate this procedure.

[14] M. M. Flood, "Recursive Methods and the Analysis of Time Series," Cowles Commission for Research in Economics, *Report of Fourth Annual Research Conference on Economics and Statistics*, Colorado Springs, 1938, pp. 90–92.

[15] These coefficients are shown, for the formulas presented, in Table 2.

[16] The regression deviation σ is calculated assuming $N-v$ degrees of freedom.

forecasts, by "serial deviation" μ is meant the standard deviation of the actual from the corresponding amount in the previous time interval, and by "mean deviation" s is meant the standard deviation of the actual from the mean of the series in the *data* period, then the ratios σ/μ and σ/s measure the corresponding reductions of variance. For short-term forecasts μ is generally less than s for economic time series and in this case the ratio σ/μ provides the more severe test of the forecasting formula. These several deviations are shown in Table 2 above for the 37 formulas presented there, and are shown separately for the data periods and the testing periods.

For the Westinghouse Orders Series, formulas 5 and 9 were applied and the resulting forecasts are shown graphically in Figure 1. The result of applying formulas 26 and 31 to forecast the Gross Sales Tax

TABLE 5

REDUCTION OF VARIANCE BY CERTAIN FORECASTING FORMULAS

Formula No.	Months Forecast	Data			Testing			Testing	
		σ	μ	s	σ	μ	s	σ/μ	σ/s
6	3	—	115	459	156	203	520	0.77	0.30
5	3	71	115	459	138	203	520	0.68	0.27
9	6	142	213	459	326	386	520	0.84	0.63
10	9	225	302	459	534	511	520	1.05	1.03
11	12	328	381	459	641	640	520	1.00	1.23
26	3	23	44	131	31	46	291	0.67	0.11
31	6	47	86	141	75	87	291	0.86	0.26

series is also shown in Figure 2. An inspection of these curves shows that the formulas yield reasonably good estimates and "catch the turning points" fairly well. More precise information concerning the forecasting accuracy of these, and of some of the other formulas used, is available on inspection of the deviations exhibited in Table 5.[17]

The deviation ratios σ/μ and σ/s show a significant reduction of variance for forecast intervals of three and six months in each case. For nine-month forecasts the deviations are about the same and for prediction periods greater than nine months the mean of the series provides estimates about as accurate as any of those considered. These general conclusions do not necessarily apply to other economic time series, of course.

Other tests of reliability of the forecasting formula were used. One such is the comparison of the serial coefficients and the autoregression coefficients derived from the formula. Another is the comparison of the formulas obtained by recursion with those obtained by regression

[17] A more detailed description of these formulas is given in Table 2.

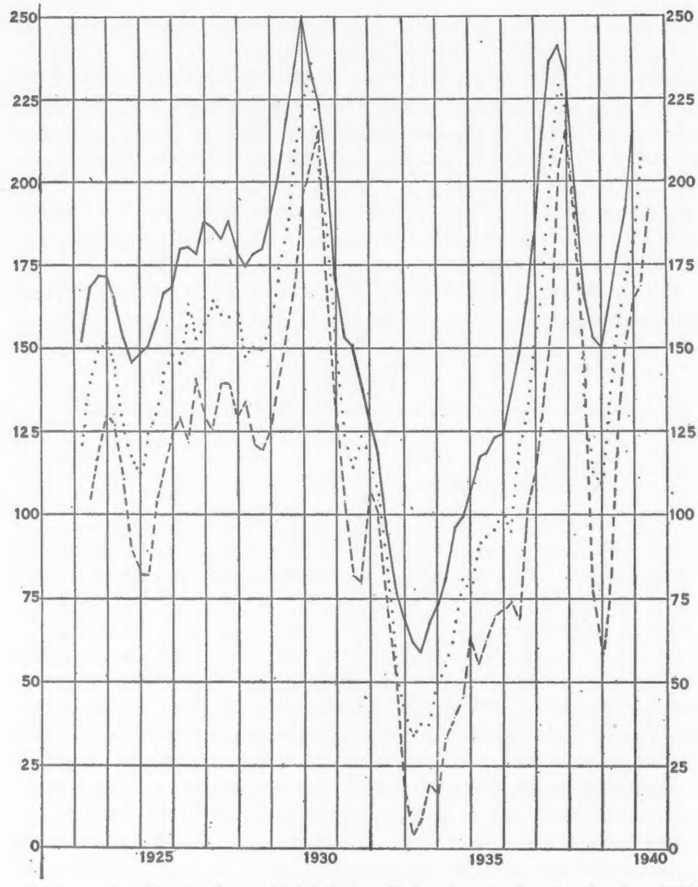

FIGURE 1.—Westinghouse Total Orders Entered, actual and estimates, 1923–1940.

———: Actual annual orders by quarters;

. . . : Estimated annual orders by quarters, estimated one quarter ahead (less $25 million);

– – – : Estimated annual orders by quarters, estimated two quarters ahead (less $50 million).

analysis, or other means. Yet another empirical test is afforded by a comparison of the difference equations obtained as the data period is lengthened; this includes the comparison of actual regression coefficients, exponential-trend exponents, and the cycles.

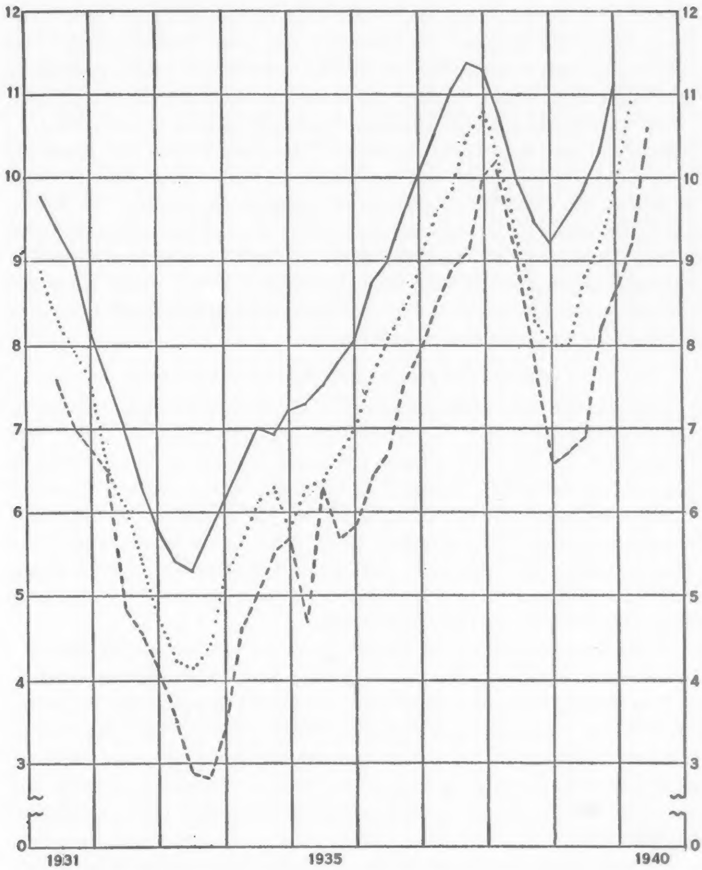

FIGURE 2.—West Virginia Article 13 Gross Sales Tax Receipts, actual and estimates, 1931–1940

———: Actual annual receipts by quarters;

. . . : Estimated annual receipts by quarters, estimated one quarter ahead (less $1 million);

– – – : Estimated annual receipts by quarters, estimated two quarters ahead (less $2 million).

Space does not permit a full discussion of the use of other tests of this type, nor do the results obtained with them warrant much attention. A comparison of the coefficients of formulas 33–37 obtained by

recursion from formulas 23–27 with the corresponding coefficients of formulas 28–32 obtained by direct least-squares fit shows very close agreement between the two sets. When overlapping data periods of only slightly different lengths were used the resulting difference equations were usually quite different. There did appear to be some persistency of period among the several formulas and some of these are shown in Table 1 given above. There are indications that at least a fifth-order difference equation is required to explain this series, probably so that the solution may have the approximate nine- and forty-month periods. This conclusion is based on general observation rather than on specific test; for example, the higher order is not required to achieve reduction of variance comparable with that of a lower-order equation in most instances.

6. THE HYPOTHESIS OF CONSTANT COEFFICIENTS

Perhaps the most difficult question of all concerns the number of observations necessary for a satisfactory data period. Yule[18] has pointed out some of the dangers when the data period is too short. If the series is subject to "shocks," at infrequent intervals, which merely change the true initial points but leave the coefficients of the difference equation constant, then a longer period is probably better, but, if the shocks change the coefficients, then a short data period, moved along, would reveal these changes and permit an examination of the changing character of these coefficients with time.

Table 6 which follows shows the extreme variation among the sets of difference equations derived from data periods of different lengths. It is extremely difficult to determine the extent to which this variation is due to the changing character of the difference equation with time as opposed to chance variation caused by the few degrees of freedom available or is due simply to the fact that the system is actually not linear or stationary, or the sample taken from a normal population as assumed.

If the several assumptions made about the system were correct then it would be reasonable to expect the coefficients of the difference equations obtained by adding additional observations to approach their correct limiting values. After determination of a number N' of observations adequate for the determination of the parameters with the desired accuracy, the next step is the examination of the changing character of the coefficients with time. This might be studied by estimating the parameters by successive sets of data periods, each with N' observations but moved along one time unit. The empirical examination of these questions is exceedingly laborious and only a brief, and some-

[18] G. U. Yule, *loc. cit.*

TABLE 6

REGRESSION EQUATIONS DERIVED FROM OVERLAPPING DATA PERIODS OF DIFFERENT LENGTHS

Formula No.		Data Period				Westinghouse Orders (12-month series)						
						Coefficients of						
No. of Coeff.	Deg. of Freed.	Time Int. ( )–12/36	Regr. St. Dev. σ	Serial St. Dev. μ	Ratio σ/μ	$T-1$	$T-2$	$T-3$	$T-4$	$T-5$	$T-6$	$T-7$
7	3	9/34	64.2	106.1	0.61	0.843	0.229	0.051	−0.668	0.397	−0.010	0.327
7	7	9/33	56.2	105.0	0.54	1.151	0.150	−0.010	−0.727	0.468	0.065	−0.029
7	11	9/32	55.1	104.1	0.53	1.471	−0.088	−0.082	−0.667	0.168	0.426	−0.196
7	13	3/32	61.4	115.0	0.53	1.641	−0.221	−0.297	−0.767	0.585	0.408	−0.367
7	23	9/29	79.9	135.2	0.59	1.762	−0.621	0.043	−0.761	0.499	0.428	−0.359
6	4	9/34	59.9	106.1	0.56	1.011	0.256	−0.211	−0.625	0.445	0.247	.....
6	8	9/33	52.6	105.0	0.50	1.140	0.142	0.022	−0.727	0.473	0.020	.....
6	12	9/32	54.9	104.1	0.53	1.512	−0.252	0.118	−0.705	0.253	0.103	.....
6	14	3/32	66.5	115.0	0.58	1.763	−0.584	0.116	−0.929	0.912	−0.265	.....
6	24	9/29	84.0	135.2	0.62	1.829	−0.883	0.324	−0.864	0.822	−0.237	.....
5	1	9/35	74.7	125.6	0.59	1.578	−0.122	−0.912	0.307	0.211	.....	.....
5	5	9/34	55.1	106.1	0.52	1.151	0.077	−0.217	−0.575	0.664	.....	.....
5	9	9/33	49.6	105.0	0.47	1.151	0.122	0.020	−0.732	0.508	.....	.....
5	13	9/32	53.2	104.1	0.51	1.566	−0.348	0.133	−0.741	0.419	.....	.....
5	14	6/32	66.8	115.8	0.58	1.617	−0.328	0.143	−0.968	0.553	.....	.....
5	15	3/32	68.0	115.0	0.59	1.659	−0.339	−0.005	−0.767	0.461	.....	.....
5	25	9/29	84.8	135.2	0.63	1.743	−0.738	0.268	−0.686	0.404	.....	.....
5	51	3/23	71.0	114.4	0.62	1.704	−0.636	0.266	−0.841	0.507	.....	.....
4	2	9/35	53.5	125.6	0.43	1.672	−0.185	−0.893	0.449	.....	.....	.....
4	6	9/34	65.9	106.1	0.62	1.114	0.088	−0.136	−0.003	.....	.....	.....
4	10	9/33	61.7	105.0	0.59	1.101	0.001	0.053	0.031	.....	.....	.....
4	14	9/32	60.8	104.1	0.58	1.521	−0.375	−0.012	−0.099	.....	.....	.....
4	15	6/32	77.6	115.8	0.67	1.567	−0.361	−0.059	−0.128	.....	.....	.....
4	16	3/32	75.5	115.0	0.66	1.586	−0.363	−0.102	−0.104	.....	.....	.....
4	26	9/29	90.9	135.2	0.67	1.771	−0.768	−0.047	0.034	.....	.....	.....
4	52	3/23	81.4	114.4	0.71	1.713	−0.665	−0.064	0.014	.....	.....	.....
3	3	9/35	51.3	125.6	0.41	1.626	−0.107	−0.498	.....	.....	.....	.....
3	7	9/34	61.0	106.1	0.57	1.114	0.088	−0.140	.....	.....	.....	.....
3	11	9/33	58.8	105.0	0.56	1.098	−0.006	−0.012	.....	.....	.....	.....
3	15	9/32	59.2	104.1	0.57	1.545	−0.345	−0.166	.....	.....	.....	.....
3	16	6/32	75.7	115.8	0.65	1.599	−0.322	−0.259	.....	.....	.....	.....
3	17	3/32	73.6	115.0	0.64	1.609	−0.329	−0.263	.....	.....	.....	.....
3	20	6/31	77.9	112.4	0.69	1.636	−0.458	−0.164	.....	.....	.....	.....
3	27	9/29	89.3	135.2	0.66	1.771	−0.795	0.014	.....	.....	.....	.....
3	53	3/23	80.7	114.4	0.71	1.713	−0.674	−0.041	.....	.....	.....	.....
2	4	9/35	52.4	125.6	0.42	1.656	−0.608	.....	.....	.....	.....	.....
2	8	9/34	57.6	106.1	0.54	1.114	−0.042	.....	.....	.....	.....	.....
2	12	9/33	56.4	105.0	0.54	1.099	−0.020	.....	.....	.....	.....	.....
2	16	9/32	58.7	104.1	0.56	1.664	−0.630	.....	.....	.....	.....	.....
2	17	6/32	75.9	115.8	0.66	1.791	−0.774	.....	.....	.....	.....	.....
2	18	3/32	74.0	115.0	0.64	1.808	−0.792	.....	.....	.....	.....	.....
2	21	6/31	77.2	112.4	0.69	1.771	−0.759	.....	.....	.....	.....	.....
2	28	9/29	87.7	135.2	0.65	1.761	−0.771	.....	.....	.....	.....	.....
2	54	3/23	80.0	114.4	0.70	1.742	−0.744	.....	.....	.....	.....	.....

what futile, attempt was made to inquire into this aspect of this problem. Further study along these lines holds some promise of success, and for that reason I shall discuss some of the difficulties in method which we encountered.

The type of empirical analysis being discussed in this paper requires the solution of a vast number of sets of linear equations of all orders. So far as I know, the most suitable method of solving such sets of normal equations is the Doolittle method, which we have used with slight variations.[19] For the particular problem mentioned in the preceding paragraph one would expect some iterative scheme to be practicable since the solution of the normal equations with one observation added, or possibly one added and one subtracted, should be about the same as for the original set. We experimented with several iterative schemes, none of which proved very satisfactory.

The most promising of the iterative schemes are of a particular type studied by Southwell and Temple.[20] If the normal equations to be solved are denoted by $Mu+c=0$, then the iteration is defined by $u_{p+1}=u_p+\lambda_p v_p$, where u_0 is the initial trial value, λ_p is a scalar, and v_p is a vector, each defined in terms of M, c, and u_p. This may be interpreted geometrically in the v-dimensional coefficient space as a point u_{p+1} obtained from the point u_p by proceeding a distance λ_p from u_p in the direction v_p. If we construct any positive single-valued function $E(M, c, u)$ defined for every u, which has a single minimum at $u=-M^{-1}c$, and such that the surfaces $E(M, u, c)=$constant are all convex, then we only need to choose a direction v_p at each trial point u_p, which is not tangent to the curve $E(M, u_p, c)=E(M, u, c)$, to be assured that a λ_p exists such that $E(M, u_p+\lambda_p v_p, c)<E(M, u_p, c)$. This iteration, if continued, must converge to the desired solution, $-M^{-1}c=\lim (u_p+\lambda_p v_p)$. Southwell and Temple[21] each use the "energy" function $(Mu+c)'M^{-1}(Mu+c)=E(M, u, c)$ and Temple chooses the "direction of steepest descent" as $v_p=Mu_p+c$, which is simply the normal to his function $E(M, u, c)$. Southwell uses v_p as a co-ordinate direction not tangent to the "energy" function. We tried other functions such as $E_j(M, u, c)=(Mu+c)'M^j(Mu+c)$ for $j\geqq 0$, with the corresponding normal direction v in each case, but none of these converges with sufficient rapidity. I review these negative results because I believe the problem is of considerable importance and the lack of a suitable iterative method has prevented us from examining our series

[19] The method is due to A. C. Aitken and is described in some detail in a paper by L. K. Tucker, *Psychometrika*, Vol. 3, 1938, pp. 189–197.

[20] G. Temple, *Proceedings, Royal Society of London*, Vol. 169A, 1939, pp. 476–500.

[21] *Ibid.*

empirically for "shocks" as we should like to do. The result is that we have, of necessity, used a difference equation with coefficients assumed independent of time.

7. TREND AND SEASONAL VARIATION

Throughout the analysis, little or no attempt was made to remove "trends" from the data. The real roots of the characteristic equation of the difference equation provide trends directly, where such trends exist. In most instances the damping was so strong that as the forecasts were extended over greater intervals about the only effect left was the exponential trend. Seasonal influences also were treated directly by assuming that they merely introduce additional short cycles. In one instance an attempt was made to isolate the seasonal factor directly as follows:

There is a very apparent seasonal variation in the quarterly receipts under the West Virginia Article 13 Gross Sales Tax. If $G(T)$ denotes the collections from this tax during quarter T we may consider, for $i=0, 1, 2, 3$, the stochastical difference equations

$$G(T + i) = \sum_{\alpha=1}^{5} a_\alpha G(T - \alpha) + \sum_{\beta=1}^{4} g_\beta e_\beta(T + i),$$

where

$$e_\beta(T) = \begin{cases} 0 \text{ if } T \not\equiv \beta \pmod 4, \\ 1 \text{ if } T \equiv \beta \pmod 4. \end{cases}$$

This assumes that the deviations $G(T)-g_1e_1(T)$, using the first quarter of each year for example, satisfy the same difference equation as do the other differences $G(T+i)-g_\beta e_\beta(T+i)$. The result of applying this method to obtain "seasonal" forecasting formulas for $G(T)$ is shown in Table 7. The first quarter of the fiscal year (ending September) is represented by $T\equiv 1 \pmod 4$. Thus the constants tabulated under $e_1(T+i)$ measure the first-quarter "seasonal," those under $e_2(T+i)$ the second-quarter "seasonal," and so on. From this table it is apparent that the addition of the $e_\beta(T+i)$ terms has resulted in a substantial reduction of variance in each of the four cases considered, whether the serial deviation or the mean deviation is used in this comparison, and for both the test and data periods.

8. THE HYPOTHESIS OF LINEARITY

Perhaps the most reliable test of the *adequacy* of a particular forecasting formula is provided by its coefficient of variability. Whether or not the forecasting formula is the best possible, it does or does not

TABLE 7
SEASONAL FORMULAS FOR FORECASTING
(West Virginia Article 13 Gross Sales Tax)

Formulas for		Degrees of Freedom	Coefficients of								
			$G(T-1)$	$G(T-2)$	$G(T-3)$	$G(T-4)$	$G(T-5)$	$e_1(T+i)$	$e_2(T+i)$	$e_3(T+i)$	$e_4(T+i)$
$G(T)$	1	17	0.81397	0.24468	0.10298	0.14170	−0.44616	1.46113	7.52560	1.67671	1.49662
	2	21	0.97151	0.03844	−0.04048	0.84789	−0.80916				
$G(T+1)$	1	17	0.89767	0.33719	0.21407	−0.34477	−0.35355	2.37569	8.57193	7.72592	2.71187
	2	21	0.94106	0.04254	0.79640	−0.02794	−0.73833				
$G(T+2)$	1	17	1.15166	0.28520	−0.17949	−0.26426	−0.38158	3.45659	10.65255	8.87099	10.15128
	2	21	0.88159	0.91800	−0.09992	0.00049	−0.67618				
$G(T+3)$	1	17	1.18944	0.07961	−0.07849	−0.07134	−0.68878	11.57510	13.99970	11.11991	11.92757
	2	21	1.64340	0.06707	−0.08964	−0.03444	−0.55316				

DEVIATIONS

Formulas for		Months Forecast	Data Period				Testing Period					
			Time Int.	σ	μ	s	Time Int.	σ	μ	s	σ/μ	σ/s
$G(T)$	1	3	9/30–12/36	171.65	423.57	483.92	3/37–6/39	263.48	409.94	673.23	0.64	0.39
	2		9/30–12/36	205.20	423.57	483.92	3/37–6/39	296.22	409.94	673.23	0.72	0.44
$G(T+1)$	1	6	12/30– 3/37	222.06	449.51	502.60	6/37–6/39	435.68	537.93	646.82	0.81	0.67
	2		12/30– 3/37	276.77	449.51	502.60	6/37–6/39	525.29	537.93	646.82	0.98	0.81
$G(T+2)$	1	9	3/31– 6/37	283.19	507.88	492.07	9/37–6/39	567.51	572.62	623.22	0.99	0.91
	2		3/31– 6/37	343.22	507.88	492.07	9/37–6/39	701.70	572.62	623.22	1.23	1.13
$G(T+3)$	1	12	6/31– 9/37	335.12	442.90	511.12	12/37–6/39	603.04	455.90	588.19	1.32	1.03
	2		6/31– 9/37	396.55	442.90	511.12	12/37–6/39	776.72	455.90	588.19	1.70	1.32

serve its immediate purpose according as this coefficient is or is not small enough. There remains the important question of the reliability of this coefficient.

Comparisons made between the data variance and the testing variance for a number of the formulas studied indicate that the data variance is often unreliable. A comparison of the deviations, or of the coefficients of variability, as given in Table 2 shown above, illustrates the extreme variation between these statistics in the data and test periods. Where these variances are found to agree, and to agree also with the recursive variance, there is empirical evidence that the formula results from consistent assumptions concerning the nature of the series being studied. If, in addition, the forecasting formulas obtained by recursion compare well with those obtained by direct regression analysis, there is further evidence of the consistency of the system. A forecasting formula with these consistency properties can reasonably be assumed to yield about as accurate forecasts, using only a linear difference equation, as is possible for the particular series studied. The forecasting formulas shown in Table 2 as given above for the West Virginia Article 13 Gross Sales Tax seem to have these consistency properties. Not only do the coefficients of variability in the data period agree with those in the test period but the coefficients obtained by recursion agree well with those obtained by regression. Further accuracy would probably be obtained only by using a different form of difference equation, or by assuming a different probability distribution.

Where it is desired only to obtain the most accurate *linear* difference forecasting equation there is no particular reason why the autoregression coefficients should agree with the serial coefficients. If they do agree, the indication is that the system is actually linear. If the system is not linear, it might actually be unwise to use the linear equation whose autoregression coefficients agree most closely with the observed serial coefficients.

In the case of a periodic function it is clear that the direct regression method gives the best linear approximation of given order to the true difference equation and that it may be considerably better than the one obtained by the method of serial coefficients. This is true whether the true difference equation is linear, but of a higher order than that used in the approximation, or whether it is actually nonlinear. It may be of interest to consider an example which is not periodic and which does not satisfy a linear difference equation, say the function $f(t) = \sin t/t$ for t intervals of length $\pi/6$.

In order to compare these two methods of linear approximation it is necessary to choose a data period and a test period for the "time series" $f(t)$. For this example let the data period include the values of

$f(t)$ for $t=0, 1, \cdots, 13$, let the test period include those for $t=14, 15, \cdots, 26$, and let the difference equation be of the form $f(t) = a_1f(t-1)+a_2f(t-2)$. A comparison of the method of serial coefficients with the regression method when applied to these data is shown in Table 8.

TABLE 8

COMPARISON OF LINEAR DIFFERENCE EQUATIONS OBTAINED BY THE METHODS OF SERIAL COEFFICIENTS AND REGRESSION FOR THE "TIME SERIES" $f(t)$

Method	a_1	a_2	σ	μ	σ/μ	Cycles
Regression	1.623	−0.743	0.0104	0.0361	0.29	9.5
Serial Coef.	2.149	−1.201	0.0229	0.0361	0.63	16.6

These results show that the regression method is much superior, in this case, to the method of serial coefficients.

9. THE CHOICE OF INITIAL CONDITIONS

Attention has been directed thus far to the forecasting difference equation, its empirical determination, and its validity in actual application. Predictions were obtained by applying this difference equation, using the most recent observations as initial points. This choice of initial points is not necessarily the best. One alternative is to choose a set of initial conditions based on recent past experience but using more than the bare quota essential for such a determination.

If we let W_0 denote this vector of initial points, and $W_t=MW_{t-1}$ denote the linear difference equation as before, then W_0 is easily determined to be,[22] in the sense of least squares,

$$W_0 = L^{-1}L_0$$

where

$$L = \sum_{\alpha=1}^{N_0} \sum_{j,k=1}^{v} (e_1'M^\alpha e_j)(e_1'M^\alpha e_k)e_{jk},$$

$$L_0 = \sum_{\alpha=1}^{N_0} \sum_{k=1}^{v} W(\alpha)(e_1'M^\alpha e_k)e_k,$$

and $N_0 \geqq v$ is the number of observations used in the determination of W_0.

This method of determining initial points was applied in the case of formula 5 for Westinghouse Total Orders Entered as shown in Table 2 above. The best initial points were determined, using this difference equation and various intervals of data, each including ten observa-

[22] M. M. Flood, *loc. cit.*

tions. These initial conditions are tabulated and compared with the five actual observations in Table 9.

TABLE 9

COMPARISON OF ACTUAL AND FORECASTED AMOUNTS RESULTING FROM VARIOUS INITIAL CONDITIONS

Amount for Year Ending	Actually Observed	Period Used in Determination of Initial Points				
		9/34–12/36	3/32–6/34	9/29–12/31	3/27–6/29	3/37–6/39
12/36	1833	1504	1334	2141	2305	2337
9/36	1631	1650	1588	1834	2283	1980
6/36	1483	1813	1725	1115	2452	1497
3/36	1360	2558	1564	614	2898	1403
12/35	1242	2430	1549	180	2440	678
Coeff. of Variability	15.0	25.4	56.1	21.6	18.6	7.0

From this table it is apparent that, in this instance, the actual observations serve as the most satisfactory initial points. The best possible initial points are shown in the last column where it is seen that, with this choice, there is a solution of the difference equation which fits the series in the prediction period very closely indeed. The tentative conclusion is that the difference equation is quite satisfactory but the methods of determining initial points need to be improved.

Similar objections may be raised in connection with the method suggested for deriving the difference equation by assuming that the first v of its autoregression coefficients coincide with the first v observed serial coefficients. Where there is reason to believe that the system actually is linear, it would probably be better to fit the autoregression coefficients to all, or almost all, of the serial coefficients. We have done little of this since it seems that these methods cannot be expected to yield the most accurate forecasting formula when the system is not linear, and our problem is the very practical one of estimating revenue yields.

Another question of this kind is raised in connection with the recursive method. If we denote the formulas obtained by regression analysis as

$$W_{t+1} = M_k W_t,$$

in contrast with those obtained from M_1 by recursion, then in a consistent system $M_k = M^k$. Instead of M_1 we might require the matrix M_1' whose powers $M_1'^k$ provide the best approximations to all the M_k, or to the first v_0 of them. I merely mention this possibility for we

have made no effort as yet to carry this analysis through, particularly since the algebra involved may be expected to present more difficulty than the practical importance of the result would seem to warrant.

10. GENERALIZATION TO A SYSTEM OF EQUATIONS

The approach which has been discussed in this paper is based essentially on the use of a single linear stochastical difference equation. A simple generalization is based on a system of linear stochastical difference equations. Instead of the single series $W(T)$ we may consider the system ${}_jW(T)$. The corresponding system of difference equations is, then,

$${}_jW(T+1) = \sum_{h=1}^{s} \sum_{k_j=1}^{v_j} ({}_h a_{k_j})({}_h W(T)).$$

These, again, may be written in the form

$$W(T+1) = MW(T)$$

and the previous discussion of a single equation generalizes immediately for such a system.[23]

In several recent theories "complete" economic systems have been constructed,[24] some of them of this multiseries type. This method was applied in the case of four economic series discussed by Newbury[25] and used by him in his forecasting method. These series are:

1. Westinghouse Total Orders Entered,
2. Total Building Construction,
3. Total U. S. Exports,
4. Amount of New Capital Issues.

A remarkably good fit was obtained in the data period used but it failed completely in the prediction period. Mr. Newbury found a similar situation when he had completed his analysis, I believe, but was able to correct the theory by adding a fifth series, Federal Deficit Spending. We have not yet extended this analysis but hope to do so in the near future. Presentation of these results in detail will be postponed until this further analysis is completed.

11. THE QUESTION OF PERIODICITIES

For some applications, it is more important to have reliable information concerning the turning points in the business cycle than to have accurate estimates of the magnitude of these swings. For this reason the forecasting formulas obtained by our various methods were ana-

[23] M. M. Flood, *loc. cit.*
[24] J. Tinbergen, *loc. cit.*
[25] F. D. Newbury, *loc. cit.*

lyzed for possible cyclic movements. A sample of these cycles is presented in Table 1.

One of the most pressing problems in this type of time-series analysis is the determination of the probability distribution theory of periods. We have no evidence concerning the possible variation of the observed periods from the "true," without which it is difficult to determine whether two formulas differ significantly as regards these cyclic movements.

As Yule[26] has pointed out, the forecasts obtained from these linear difference equations decrease in efficiency as the prediction interval is increased. For long forecasts it may be better to use the ordinary formulas derived by harmonic analysis of the time series. I hope to examine this question at a later time for it is entirely possible that we may be able to improve our long-term forecasts in this way.

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[26] G. U. Yule, *loc. cit.*

THE EFFECT OF THE UNDISTRIBUTED PROFITS TAX UPON THE DISTRIBUTION OF CORPORATE EARNINGS; A NOTE

By Harry G. Guthmann

On reading Mr. McIntyre's article[1] on the undistributed profits tax one is chiefly impressed not that he reaches the conclusion that the tax has no appreciable effect on dividend distributions but that his chart apparently indicates that the tax actually had the effect of causing a distribution that was very much *below normal.* His scatter-chart showing the ratio of dividends to earnings on the vertical scale and the per cent of industrial activity to "normal" on the horizontal scale shows a negative relationship as would be expected. But the two data for 1936 and 1937, the two years during which the tax was in effect, while showing a relatively high distribution compared with the mass of data, are considerably *below and to the left* of this main "island" of cases. The impression conveyed to the reader is that the dividend distributions of 1936 and 1937 were really quite low when read in the light of industrial activity. Thus it appears that the undistributed profits tax has the effect of causing retention rather than distribution of earnings.

A possible explanation of this extraordinary result would lie in the definition of normal industrial activity for the years 1936 and 1937. As in most indexes of normality weighted heavily with the tendencies established in the 1920's there may well be a considerable overstatement of the normal trend line. The data which will establish current normality lie in the future and so are indeterminate. The final data for any trend or normal series are the most uncertain and the whole significance of the chart used in this article depends upon the accuracy of this normal for the last two years, 1936 and 1937. If a lower normal for industrial activity were established, the present position of the dividends-to-earnings ratios would move to the right and so alter the picture at its crucial point.

A simpler and clearer picture is obtained by merely charting the Cowles indexes of earnings and dividends. Reference to the accompanying chart, Figure 1, showing these figures, leads one to the conclusion that the proportions of earnings distributed in 1936 and 1937 were comparatively high. Of the years in which dividends were less than earnings (and so bore a substantial relation to *that* year's earnings) only eight years showed a higher per cent than 1936 and 1937, three were within the range of the 1936 and 1937 data (74.4 per cent and 78.7 per cent), and forty-seven were lower.

[1] Econometrica, Vol. 7, October, 1939, pp. 336–348.

Of the seven years in which the dividend distributions exceeded 100 per cent, five were years in which earnings declined considerably from the preceding year; only two, 1933 and 1934, were years of rising earnings and these were years of depressed income. Whenever the distribution is over 100 per cent, it is necessarily based to some extent

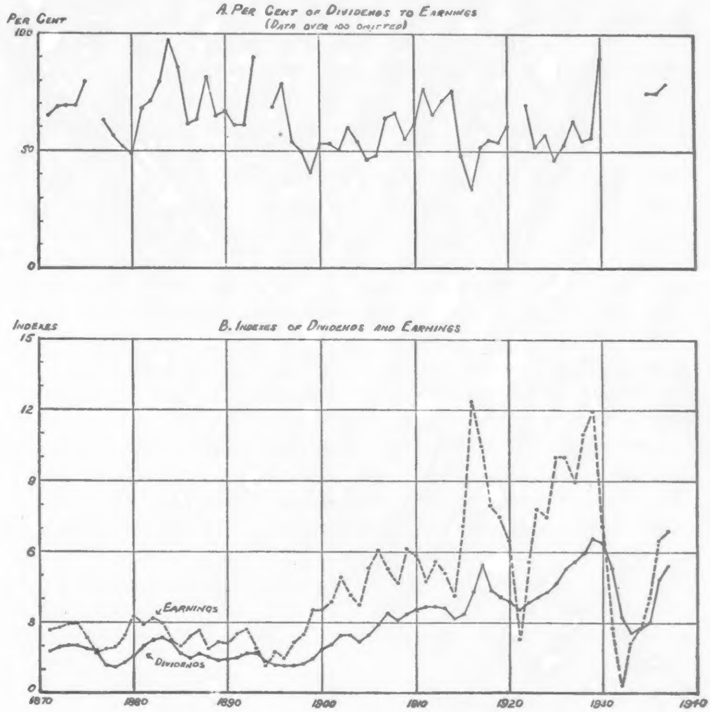

FIGURE 1.—Dividends and Earnings, 1871–1937.
Source: Cowles, *Common-Stock Indexes.*

upon the earnings of earlier periods. In all of the other years in which the percentage was above or in the range of the 1936–1937 figures, there were declining earnings, save in the three years, 1935, 1936, and 1937.

Possibly in these recent years there has been a tendency for large corporations (which this sample undoubtedly represents) to make larger distributions. As the country grows larger and reaches economic maturity the opportunities for growth tend to diminish. Furthermore,

since the World War the Federal income tax has caused corporations to show depreciation allowances that were often absent before. These noncash deductions from reported profits have placed the corporations in funds for either replacement or expansion and to that extent made earnings retention less necessary.

Whatever the reasons, this review of the figures gives one the distinct impression that the dividend distributions were decidedly high in the years 1936 and 1937. They were exceeded in only a minority of the other years and in almost all cases these were years of falling earnings. At such times dividend policy has a tendency to lag after the event of changing earnings. Only in 1935 and the two years under scrutiny do we find such a high distribution coupled with rising earnings.

But the issue for American business men is not as the article suggests a matter of whether the *aggregate* of dividends is largely affected. Since the undistributed profits tax was graduated and did not penalize heavily a moderate amount of retention, it might reasonably be supposed to have had little or no effect on many companies. The trouble lay in the disrupting effect of the tax upon those corporations which needed funds badly for debt retirement (the law exempted only sinking funds specifically drawn from profits by the terms of their indenture). Furthermore, it made no allowance for corporations with large balance-sheet deficits, which would generally mean that dividends were forbidden by state law. Perhaps most important would be the situation of smaller companies which would not appear in a study of this sort at all. These smaller companies with unlisted securities generally have no access to the capital market and consequently are virtually unable to raise equity money save through the retention of profits. The continuance of a reasonably competitive society depends upon such corporations being able to survive. For such corporations Mr. Amos' thesis[2] that corporations should be compelled to resort to the capital market has no force.

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[2] *Ibid.*, p. 340.

THE EFFECT OF THE UNDISTRIBUTED PROFITS TAX; A REPLY

By Francis McIntyre

I appreciate the opportunity of replying to Professor Guthmann's gracious note. In the hope that some sources of confusion in my paper may thus be eliminated, I shall consider what I believe to be his principal contentions:

(1) That my chart[1] shows the dividend-earnings ratios for 1936 and 1937 significantly below normal; that this implies the undistributed profits tax had the effect of causing *retention* of earnings; and that this extraordinary result casts doubt on the validity of the analysis.

(2) That a better understanding of the effects of the tax is to be obtained from his time series chart of the Cowles Commission indexes of dividends and earnings; and that this chart shows the proportion of earnings distributed in 1936 and 1937 to have been exceptionally high.

(3) That from the point of view of individual business men the effect of the tax upon aggregate earnings distribution is immaterial; and that the law as drafted was inequitable, in that it placed the greatest burdens on small corporations without access to the capital market, and on corporations with large debts or deficits.

First, the chart referred to (p. 344) was presented as a simple scatter diagram and not as a regression analysis precisely because I was unwilling to assign a "normal" percentage of earnings to be distributed at each level of business activity. While a general negative relationship between the dividend-earnings ratio and activity is familiar to all students of this problem, and is borne out by the scatter diagram, the myriad factors other than business activity which must influence corporate distribution decisions would make dependence on a mathematical equation relating these two factors alone foolhardy in the extreme. While I believe industrial activity to be the most important *single* variable influencing the dividend-earnings ratio, I recognize that only a third (31 per cent actually) of the variability in the latter may be accounted for by the former.

If the years in which dividends exceeded earnings be omitted (a procedure Professor Guthmann advocates elsewhere in his note, but with which I cannot agree—cf. pp. 343, 345), the points for 1936 and 1937 lie even closer to the regression line; they are certainly neither significantly above nor below it.

[1] Econometrica, Vol. 7, No. 4, October, 1939, p. 344. All page references in parentheses are to that volume.

With Professor Guthmann's able statement of the fact that we never can know what is "normal" until long after it is too late to do us any good, I am in substantial agreement. It may be that the Cleveland Trust index does the years 1936 and 1937 an injustice by crediting them with only 83 to 85 per cent of "normal" business activity. But of this possibility too much may easily be made. I submit that no conceivable change in these estimates would shift the points for 1936 and 1937 to a position significantly above and to the right of the general scatter of observations.

Second, I cannot feel that Professor Guthmann's Figure 1, the data for which are taken from the Cowles Commission indexes,[2] leads to essentially different conclusions. Part A plots the dividend-earnings ratio against time. It thus abstracts from the effect of industrial activity upon earnings distribution, which Figure 1 (p. 344) attempts to reveal. However, it may be instructive to investigate the possibility of trend in the dividend-earnings ratio, and for this purpose Professor Guthmann's chart is, of course, preferable. I am reluctant to accept Professor Guthmann's inference (by which he justifies the omission of dividend-earnings ratios in excess of 100 per cent) that unless dividends in any year are less than earnings they bear "no substantial relation to that year's earnings," but I do not believe this omission materially affects the conclusions to be drawn, either from his chart or mine. (This question is different from, but related to, that of the inclusion or omission of individual corporations which paid dividends in excess of earnings. On this point, cf. pp. 343, 345.)

What then are the conclusions to be drawn from Part A? They will vary with the observer, no doubt, but I may offer a few which are not adverse to the thesis of my earlier remarks.

Guided by Professor Guthmann's own suggestion to this effect,[3] we may note a rising trend in the dividend-earnings ratio since 1915. If it be legitimate to extrapolate this trend through 1937, the dividend-earnings ratios for 1936 and 1937 would certainly not appear excessively high. Two graphic objections to this extrapolation suggest themselves: (a) The four year gap (1931–34) is a wide one to be bridged in this way. (b) The sharp change between 1914 and 1915 indicates that the rather clearly defined trend of 1898–1914 could not survive the first World War. Perhaps the financial wars of the first part of the last decade similarly destroyed the 1915–1930 trend. On the other hand, the ratio for 1935 is consistent with the extrapolation, and

[2] Alfred Cowles 3rd and Associates, *Common-Stock Indexes, 1871–1937*, Bloomington, Principia Press, 1938.

[3] Pp. 355–356, *supra*.

it is surely unaffected by the tax on undistributed profits, which was enacted in 1936.[4]

Considerable weight must indeed be attached, in my opinion, to the fact that the ratio of dividends to earnings in 1935 so closely approximates that for the two following years, when the tax was in effect. The argument may be advanced that, since 1936 and 1937 were more prosperous than 1935, we should expect a decline in the ratio; the fact that a decline did not occur must therefore be accepted as proof that the ratio was higher than it would have been without the tax. This oversimplifies the case. The reported dividends for 1936 represent almost five quarters' distribution, since tax credits could be obtained only on the earnings distributed in the same calendar year; the custom of paying fourth-quarter dividends in January or February of the following year had therefore to be abandoned (cf. pp. 347–348). This is an important factor when it comes in a period of rising earnings. Further, not all of 1937 was more prosperous than 1935. Many of those corporations which distributed 50 to 75 per cent of their earnings for the first two or three quarters of that year found in December that their losses in the fourth quarter had the effect of making their 1937 dividends in excess of 1937 earnings, even if no fourth-quarter declaration were made. The high annual ratio of dividends to earnings to which this development gave rise must not be attributed to the undistributed profits tax.

Third, and finally, the paper under review was an effort to observe the effect of specific legislation upon total corporate dividend vs. reinvestment policy. The social significance of this tax can scarcely be measured (if at all) without consideration of its aggregate (i.e., net) effect on the diversion of purchasing power from one group of persons to another. Its effect upon individual entrepreneurs (perhaps even upon "competitive society") may be so important as to merit extensive and intensive study if the legislation is proposed again. Suffice it to say that such was not my purpose. I make no brief for (or against) the undistributed profits tax. I do not regard anything I said in the earlier paper or in this reply as constituting a defence of the law. Neither its iniquities nor its inequities were considered; this must not be taken as a denial of their existence. The fact that the corporations

[4] The ratios for 1936 and 1937 are also consistent with the extrapolation of this trend, *on the assumption that the tax did not effect the ratio.* This assumption, of course, I may not make, since to do so would be to justify the extrapolation by the position of these two points, and then to conclude that the proximity of these points to the extrapolated trend demonstrated the negligible effect of the tax.

sampled are the largest is a valid criticism if the effect upon small enterprises is sought; it would appear to have less weight in the determination of aggregate effect, unless it can be shown that the bulk of corporate saving is done by the small corporations not sampled by such a study.

Indiana University

ANNOUNCEMENT OF THE NEW ORLEANS AND CHICAGO MEETINGS, DECEMBER, 1940

December meetings of the Econometric Society will be held this year in both New Orleans, Louisiana, and Chicago, Illinois. At New Orleans, where joint sessions are planned with the American Economic Association during its meeting from Friday, December 27, to Monday, December 30, inclusive, the primary emphasis of the Econometric Society's program will be on Economic Theory. At Chicago, where joint sessions are planned with the American Statistical Association during its meeting from Thursday, December 26, to Saturday, December 28, inclusive, its program will emphasize Statistics.

A detailed program will be sent to each member of the Society in the United States and Canada early in December. Any inquiries regarding these sessions should be addressed to Francis McIntyre, Secretary of the Program Committee, The Econometric Society, Indiana University, Bloomington, Indiana.

BACK NUMBERS OF ECONOMETRICA

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Copies of other issues may be obtained at the list price of $1.75 ($1.50 to members), except Volume 5, Number 1, January, 1937, the price of which will be $2.50 per copy.

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SUMMARY

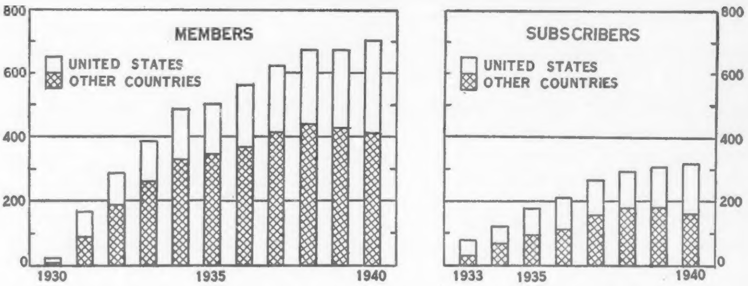
Members, October 1, 1939		671
New Members Added		86
		757
Less: Deaths	3	
Resignations	28	
Dropped for nonpayment of dues	24	55
Members, October 1, 1940		702

OBITUARY

ECONOMETRICA records with deep regret that the deaths of the following members of the Econometric Society have been reported during the past year.

PROFESSOR P. BONINSEGNI
MR. C. F. HIRSHFIELD
PROFESSOR SVEN DAG WICKSELL

Growth and Distribution of Membership in the Econometric Society and of Subscribers to Econometrica



Geographic Distribution of Members and Subscribers

Country	Members	Subscrib-ers	Total	Country	Members	Subscrib-ers	Total
United States	299	159	458	Egypt	3	1	4
England	51	17	68	Chile	3	....	3
Italy	44	21	65	Neth. India	1	2	3
France	46	8	54	Yugoslavia	3	....	3
Netherlands	33	6	39	Burma	1	1	2
Japan	13	25	38	Estonia	1	1	2
Germany	23	11	34	Philippines	1	1	2
Norway	24	3	27	Spain	2	....	2
Sweden	12	6	18	U. S. S. R.	....	2	2
Denmark	15	2	17	Algeria	1	....	1
Hungary	15	2	17	Belg. Congo	1	....	1
Switzerland	14	3	17	Cuba	....	1	1
Belgium	12	4	16	Greece	....	1	1
China	10	4	14	Guatemala	1	....	1
Poland	13	1	14	Latvia	....	1	1
Argentina	9	4	13	Manchuria	....	1	1
India	6	7	13	Mexico	....	1	1
Australia	4	5	9	Puerto Rico	....	1	1
Czecho-slovakia	9	....	9	Slovakia	1	....	1
Canada	3	4	7	Straits S.	....	1	1
Rumania	7	....	7	Tasmania	....	1	1
Bulgaria	5	1	6	Uruguay	1	....	1
Scotland	4	2	6	Venezuela	1	....	1
South Africa	4	2	6	Wales	....	1	1
Brazil	4	1	5				
Finland	3	2	5	Total	703	317	1020

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